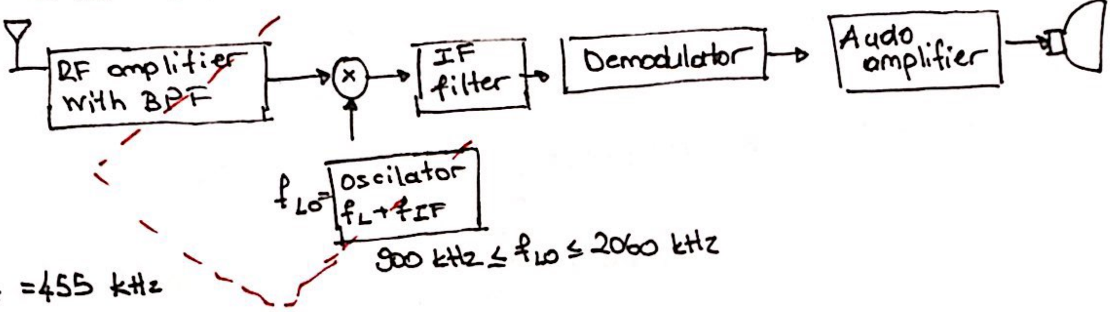
Superheterodyne AM receiver

15.10.2016
15:00-16:00
(1)

- Typical Frequency Parameters of AM and FM radio receivers

	AM	FM
RF carrier range	0.535-1.605 MHz	88-108 MHz
IF	0.455 MHz	10.7 MHz
IF bandwith	10 kHz	200 kHz

535 kHz $\leq f_{IF} \leq$ 1605 kHz



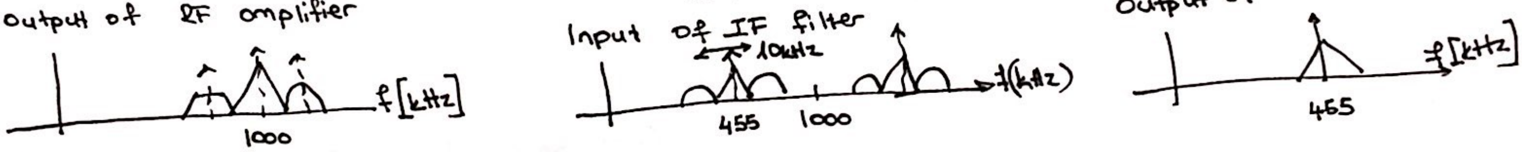
f_{IF} = 455 kHz

Example:

IF f_c = 1000 kHz

$f_{LO} = f_c + f_{IF} = 1000 + 455 = 1455$ kHz

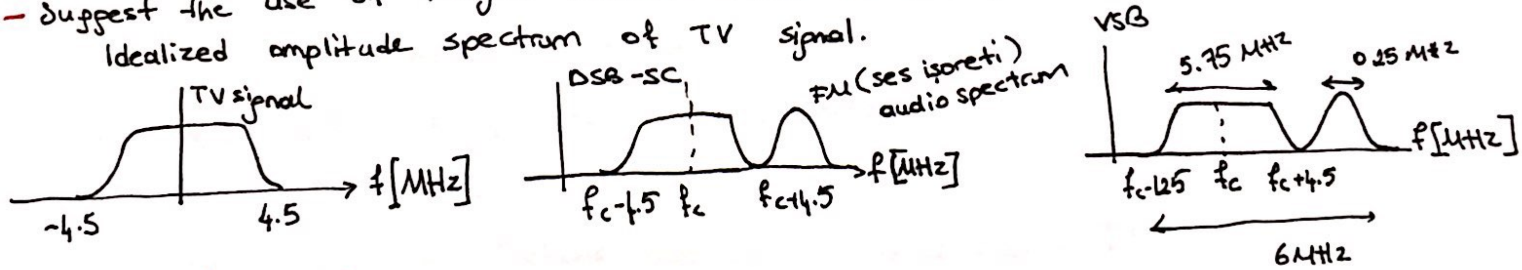
$\cos(10^6 \cdot 2\pi \cdot t)\cos(2.433 \cdot 10^6 \cdot 2\pi \cdot t)$



Television Signal

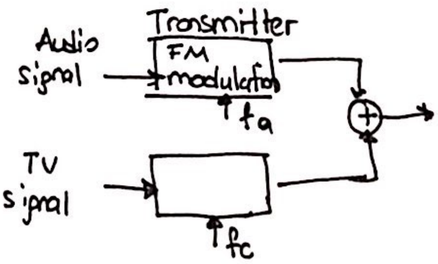
- Television signal exibits a large bandwith and significant low-frequency content.
- Television signal exibits a large bandwith and significant low-frequency content.
- Suggest the use of vestipal sideband modulation (VSB)

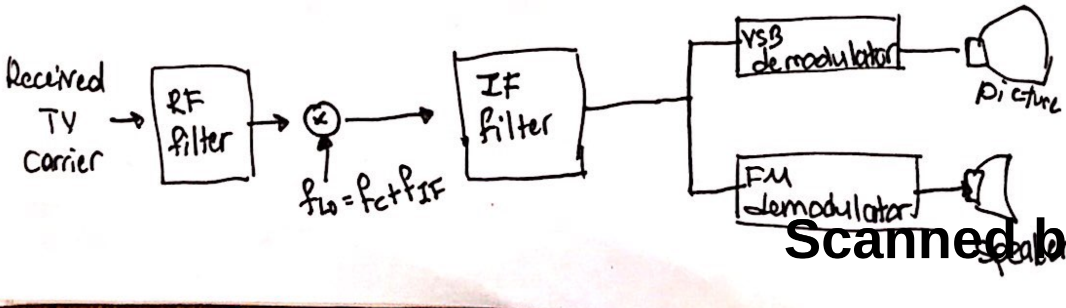
Idealized amplitude spectrum of TV signal.



TV channel Assignments

Channel number		Freq. Band [MHz]
VHF	2,3,4	54-72
VHF	5,6	76-88
UHF	14-83	470-890





Angle Modulation

- Advantage: It can provide better discrimation against noise and interference than amplitude modulation.
- Disadvantage: The cost of increased system completely

$$s(t) = A_c \cdot \cos(\theta_i(t))$$

$\theta_i(t)$: Instantanous angle

$f_i(t)$: " frequency

$$\frac{d\theta_i(t)}{dt} = 2\pi f_i(t)$$

ANGLE MODULATION:

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1) Phase Modulation (PM)

$\underbrace{\theta_i(t)}_{\text{anlık açı}} = 2\pi f_c t + k_p m(t)$
↳ taşıyıcı frekansı

m(t): message sinyal
k_p: Phase sensitivity factor

PM: $s(t) = A_c \cdot \cos(2\pi f_c t + k_p m(t))$

2) Frequency Modulation (FM)

$$\frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_i(t)$$

$$f_i(t) = f_c + k_f m(t)$$

Anlık frekansı önce anlık açıya dönüştüren

k_f: Frequency sensitivity factor

$$\theta_i(t) = 2\pi\int_0^t f_i(\tau)d\tau = 2\pi\int_0^t f_c + k_f m(\tau)d\tau = 2\pi f_c t + 2\pi k_f\int_0^t m(\tau)d\tau$$

FM: $s(t) = A_c \cdot \cos(2\pi f_c t + 2\pi k_f) \int_0^t m(\tau)d\tau)$

Properties of Angle Modulation (Ac değişmediğinden güç sbt kalcak.)

1. Constancy of transmitted power

$$P = \frac{A_c^2}{2}$$

//AM'de güç mesaj işaretine bağlıydı yan bandlardan gece

2. Doğrusal değil. $\cos(\underbrace{2\pi f_c t + k_p m(t)}_{\text{lineer değil.}})$

Nonlinearity of modulation process

$m_1(t) \rightarrow s_1(t) = A_c \cdot \cos(2\pi f_c t + k_p m_1(t))$

$m_2(t) \rightarrow S_2(t) = A_c \cdot \cos(2\pi f_c t + k_p m_2(t))$

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$m_1(t) + m_2(t) \rightarrow s(t) = A_c \cdot \cos(2\pi f_c t + k_p(m_1(t) + m_2(t)))$

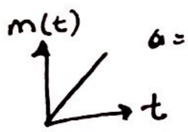
$s(t) \neq s_1(t) + S_2(t)$ "Nonlinear"

Faz modulasyonu ve frekans modülasyonu → LİNEER değil

. genlik modulasyonunda dardoleside bandda lineer.

3. Irregularity of zero-crossing

Example: $m(t) = \begin{cases} at, & t \geq 0 \\ 0, & t < 0 \end{cases}$



$a = 1$ volt/s

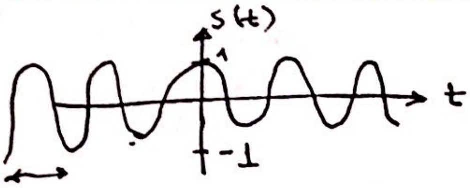
$f_c = \frac{1}{4}$ Hz

$A_c = 1$ volt
$k_p = \frac{\pi}{2}$ rad/volt olsun.

PM: phase modulation

$s(t) = \begin{cases} A_c\cos(2\pi f_c t + k_p a t), & t \geq 0 \\ A_c\cos(2\pi f_c t), & t < 0 \end{cases}$

m(t)=0
Girişi yok.



0'ı kestiği noktalar değişiyor.

$k_p = \frac{\pi}{2} \rightarrow 2\pi$'de bir kendini tekrarlar

FM: $t \geq 0$

$$\theta_i(t) = 2\pi \int_0^t f_c + k_f m(\tau) d\tau = 2\pi f_c t + 2\pi k_f \int_0^t a\tau d\tau = 2\pi f_c t + \pi k_f a t^2$$

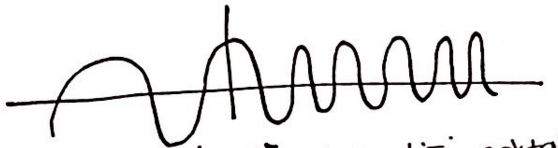
$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + \pi k_f a t^2), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

$t<0 \rightarrow m(\tau) = 0$

$$\theta_i(t) = 2\pi \int_0^t f_c = 2\pi f_c t$$

$k_f = 1$ Hz/volt

$A_c = 1$ volt olsun.



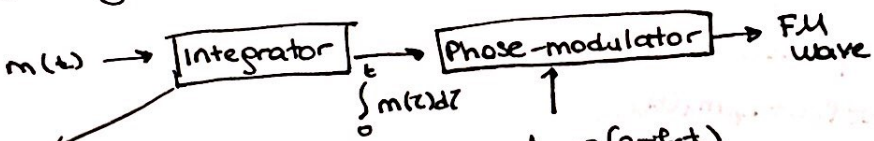
0'ı kestiği noktalar değişiyor.

4. Visualization difficulty of message waveform → Burda $s(t)$'ye bakarak $m(t)$'yi anlamak zor. Genlik mod.'nun zarfı mesaj işaretini veriyordu.

5. Trade off increased transmission bandwidth for improved noise performance.

Relation between PM and FM waves

— Generating an FM wave by using a phase modulator



mesaj işaretinin integralini alıyor.

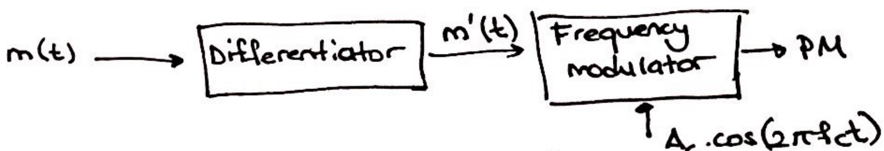
$$s(t) = A_c \cdot \cos\left(2\pi f_c t + k_p \int_0^t m(\tau) d\tau\right)$$

• Bir integratör ve faz modulatör ile FM işaretini üretirsin

— Generating a PM wave by using a frequency modulator.

FM mesaj işaretinin integralini alır. Frekans modulatör.

O zaman onun girdisine türevini alınmış gir ki sonunda $m(t)$'yi versin.



$$s(t) = A_c \cdot \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m'(\tau) d\tau\right) = A_c \cdot \cos(2\pi f_c t + 2\pi k_f m(t))$$

Summary of Angle Modulation

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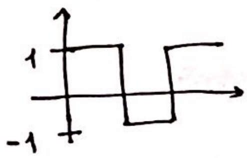
	PM	FM
Instantaneous phase $\theta_i(t)$	$2\pi f_c t + k_p m(t)$	$2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau$
Instantaneous freq. $f_i(t)$	$f_c + \frac{k_p}{2\pi} m'(t)$	$f_c + k_f m(t)$
modulated wave	$A_c \cdot \cos(2\pi f_c t + k_p m(t))$	$A_c \cdot \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$

Example: FM



$k_f = 10^5$

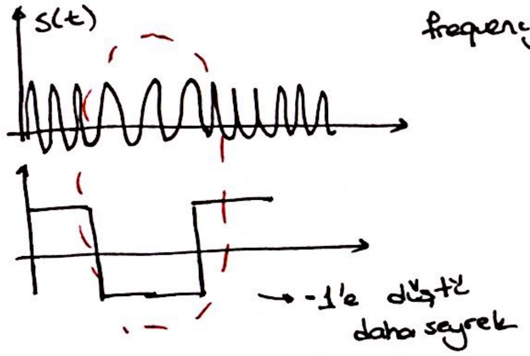
$f_c = 100$ MHz

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau)d\tau = 2\pi \int_0^t f_c + k_f m(\tau)d\tau = 2\pi f_c t + 2\pi f_c t \int_0^t m(\tau)d\tau$$

$m(t) = 1 \qquad \theta_i(t) = 2\pi f_c t + 2\pi k_f t = 2\pi(f_c + k_f)t$

$m(t) = -1 \qquad \theta_i(t) = 2\pi f_c t - 2\pi k_f t = 2\pi(f_c - k_f)t$

$$s(t) = \begin{cases} A_c \cdot \cos(2\pi(100{,}1 \times 10^6)t), & m(t) = 1 \\ A_c \cdot \cos(2\pi(99{,}9 \times 10^6)t), & m(t) = -1 \end{cases}$$



frequency shift keying (FSK)

Aynı örneği bu sefer PM ile yapalım.

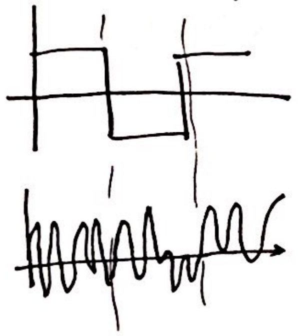
PM $\quad k_p = \frac{\pi}{2} \quad f_c = 100$ MHz

$\theta_i(t) = 2\pi f_c t + k_p m(t)$

$m(t) = 1 \qquad \theta_i(t) = 2\pi f_c t + k_p$

$m(t) = -1 \qquad \theta_i(t) = 2\pi f_c t - k_p$

$$s(t) = \begin{cases} A_c \cos(2\pi 10^8 t + \frac{\pi}{2}), & m(t) = 1 \\ A_c \cdot \cos(2\pi 10^8 t - \frac{\pi}{2}), & m(t) = -1 \end{cases} = \begin{cases} -A_c \cos(2\pi 10^8 t), & m(t) = 1 \\ A_c \cos(2\pi 10^8 t), & m(t) = -1 \end{cases}$$



phase shift keying (PSK)

Frequency modulation

We first consider the simple case of a single tone modulation that produces narrowband FM

We next consider more general case wideband FM

Narrowband FM

$m(t) = A_m \cos(2\pi f_m t)$

$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$

frequency deviaton $\Delta f = k_f \cdot \underbrace{\max}_{\text{tepe değerlerinden max olanı al}}(|m(t)|) = k_f A_m$

$f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$

$\theta_i(t) = 2\pi \int_0^t f_i(\tau)d\tau = 2\pi \int_0^t f_c + \Delta f \cos(2\pi f_m \tau)d\tau = 2\pi f_c t + \frac{2\pi \Delta f}{2\pi f_m}\sin(2\pi f_m t)$

modulation index $\beta = \frac{\Delta f}{f_m}$

$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$

FM wave: $s(t) = A_c \cdot \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

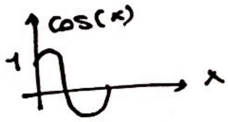
$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$s(t) = A_c \cdot \cos(2\pi f_c t)\cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t)\sin(\beta \sin(2\pi f_m t))$

Approximate form of narrowband FM

If $\underbrace{\beta \ll 1}_{\text{B'nın küçük değeri için}}$ $\Rightarrow$ sin 0'a yakın olacak.

$\sin x \approx x$

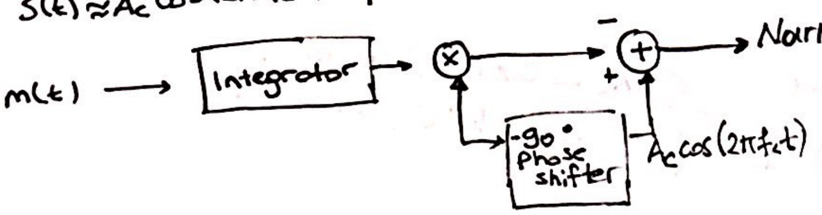


$\cos x \approx 1$

gibi düşünebilsin.

$\cos(\beta \sin(2\pi f_m t)) \approx 1$ $\quad$ $\sin(\beta \sin(2\pi f_m t)) \approx \beta \sin(2\pi f_m t)$

$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$



(normal FM de genlik sbt = A_c idi)
Burada genlik değişti.

1. The envelope contains a residual amplitude modulation that varies with time.

2. The angle contains harmonic distortion.

Narrowband FM

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$$s(t) = A_c \cdot \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$$

Polar Baseband Representation of FM

$$s(t) = S_I(t) \cdot \cos(2\pi f_c t) - S_Q(t)\sin 2\pi f_c t = a(t) \cdot \cos(2\pi f_c t + \phi(t))$$

$a(t) = \sqrt{S_I^2(t) + S_Q^2(t)}$ envelope $\quad \phi(t) = \tan^{-1}\left(\frac{S_Q(t)}{S_I(t)}\right)$ phase

Angle: $\theta(t) = 2\pi f_c t + \phi(t)$

Phase: $\phi(t) = \tan^{-1} \beta \sin(2\pi f_m t))$

For FM signal

$S_I(t) = A_c$, $\quad S_Q(t) = \beta \cdot A_c \sin(2\pi f_m t)$

Envelop: $a(t) = \sqrt{A_c^2 + \beta^2 A_c^2 \sin^2(2\pi f_m t)} = A_c \underbrace{\sqrt{1 + \beta^2 \sin^2(2\pi f_m t)}}_{\text{amplitude distortion}}$

$$\tan^{-1} x \cong x - \frac{1}{3}x^3$$

$$\phi(t) \simeq \beta \sin(2\pi f_m t) - \frac{1}{3}\beta^3 \sin^3(2\pi f_m t)$$

$$\theta(t) = 2\pi f_c t + \beta \sin(2\pi f_m t) - \underbrace{\frac{1}{3}\beta^3 \sin^3(2\pi f_m t)}_{\text{harmonic distortion}}$$

Wideband FM

FM: $s(t) = A_c \cdot \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

$s(t)$ is not periodic $\quad s(t) \neq s(t+T)$

Complex Baseband Representation of FM

$$s(t) = Re\left\{A_c \exp\{j2\pi f_c t + j\beta \sin(2\pi f_m t)\}\right\} = Re\left\{A_c \cdot e^{j\beta \sin(2\pi f_m t)} \cdot e^{j2\pi f_c t}\right\}$$

$\tilde{s}(t) = A_c \cdot e^{j\beta \sin(2\pi f_m t)}$ complex envelope

$\tilde{s}(t)$ is periodic with $T = \frac{1}{f_m}$

$$\tilde{s}(t+T) \stackrel{?}{=} \tilde{s}(t) = A_c \cdot e^{j\beta \sin(2\pi f_m (t + \frac{1}{f_m}))} = A_c \cdot e^{j\beta \sin(2\pi f_m t + 2\pi)} = A_c \cdot e^{j\beta \sin(2\pi f_m t)}$$

we many therefore expand $\tilde{s}(t)$ in the form of a complex Fourier Series

$$\tilde{s}(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{j2\pi n f_m t}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{+T/2} \tilde{s}(t) \cdot e^{-j2\pi n f_m t} dt = f_m \int_{-1/2f_m}^{+1/2f_m} A_c \cdot e^{j\beta \sin(2\pi f_m t) - j2\pi n f_m t} dt \qquad T = \frac{1}{f_m}$$

$x = 2\pi f_m t$

$dx = 2\pi f_m dt$

$$C_n = \frac{A_c}{2\pi}\int_{-\pi}^{+\pi} e^{j\beta \sin(x) - jnx} dx$$

$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{j\beta \sin x - jnx} dx \quad$ n^{th} order Bessel function of the first kind and argument β.

$$C_n = A_c J_n(\beta)$$

$$\tilde{s}(t) = A_c \cdot \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j2\pi n f_m t}$$

FM signal

$$s(t) = Re\left\{ \tilde{s}(t) e^{j2\pi f_c t} \right\} = Re\left\{ A_c \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j2\pi(f_c + n f_m)t} \right\} = A_c \sum_{n=-\infty}^{+\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

Fourier Spectrum

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{+\infty} J_n(\beta) \left[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m) \right]$$

Wideband FM

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Fourier series expansion of single tone FM signal

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

besel fonksiyonu

Fourier spectrum of s(t)

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)\right]$$

Properties of Bessel function:

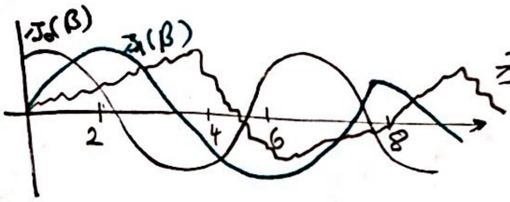
1) a) $J_n(\beta) = J_{-n}(\beta)$ for n even

b) $J(\beta) = -J_{-n}(\beta)$ for n odd

2) For small values of β

$J_0(\beta) \approx 1$ $\quad J_1(\beta) \approx \frac{\beta}{2}$

$J_n(\beta) \approx 0 \quad n > 1$



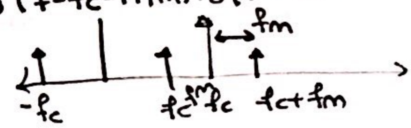
genlikleri giderek azalıyor.

3) $\sum_{n=-\infty}^{+\infty} J_n^2(\beta) = 1$

Observations

1) The spectrum of FM signal contains a carrier component and an infinite set of side frequencies

$\delta(f - f_c - n f_m) + \delta(f - f_c + n f_m)$



2) For small β only Bessel coefficients $J_0(\beta)$ and $J_1(\beta)$ have significant values

(special care – narrow band FM)

FM signal is effectively composed of a carrier a single pair of side freq. at $f_c \mp f_m$

Avarage Power of FM signal

$$P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{A_c^2}{2}$$

• FM işareti, işaretin genliğini değiştirmiyor, frekansını değiştiriyor. o yüzden güç de değişmiyor.

FM spectrum for varying amplitude (f_m sbt, A_m değişiyor)

message $\quad m(t) = A_m \cdot \cos(2\pi f_m t)$

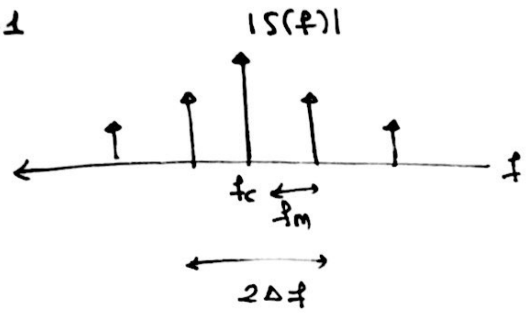
f_m is fixed

A_m is varied

Frequency deviation $\quad \Delta f = k_f \cdot A_m$

modulation index $\quad \beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$

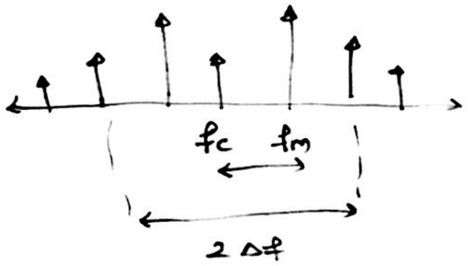
$\beta = 1$



• f büyüdükçe oklar sıfıra doğru gider.

$\Delta f = \beta f_m = f_m$

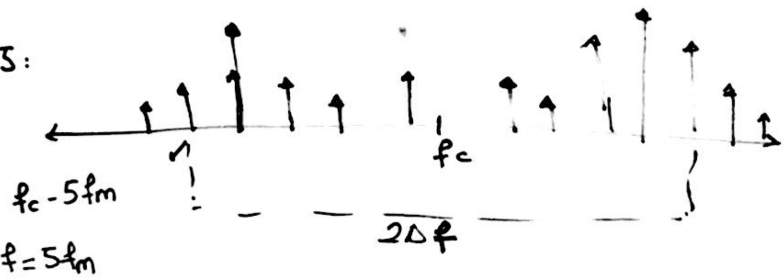
$\beta = 2$:



$\Delta f = \beta f_m = 2 f_m$

β arttıkça frekans sapması artıyor.

$\beta = 5$:



$\Delta f = 5 f_m$

FM Spectrum for varying frequency

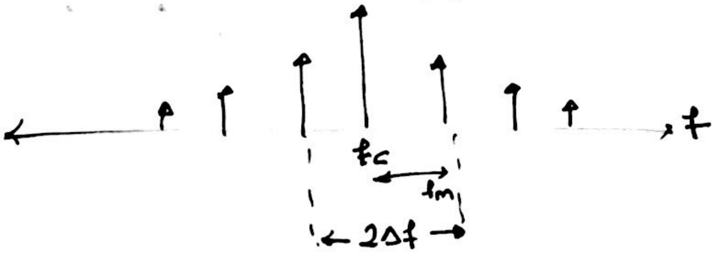
$m(t) = A_m \cdot \cos(2\pi f_m t)$

A_m is fixed

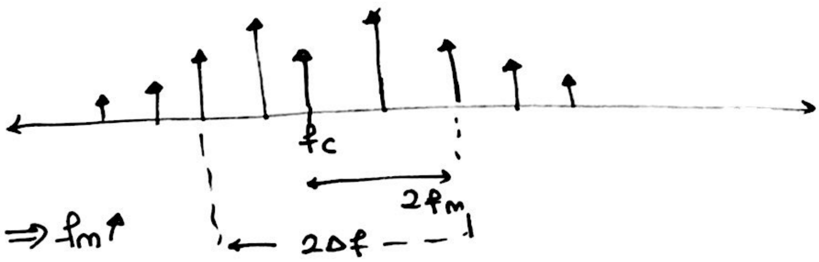
f_m is varied

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f \cdot A_m}{f_m}$$

$\beta = 1$:



$\beta = 2$:



$\beta \downarrow \Rightarrow f_m \uparrow$

Bandwidth of fm $= 2\Delta f$

Transmission Bandwidth of FM waves

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- In theory, an FM wave contains an infinite number of side freq. so that the bandwith required to transmit such a modulated wave is similary infinite in extand.
- In practise, however, we find that the FM wave is effectively limited to a finite number of significant side frequencies
- For large values of $\beta \to \infty$, bandwith of FM $\to 2\Delta f$
- For small values of β, spectrum of FM is limited to the carrier freq. and one pair of side frequencies at $f_c \mp f_m$

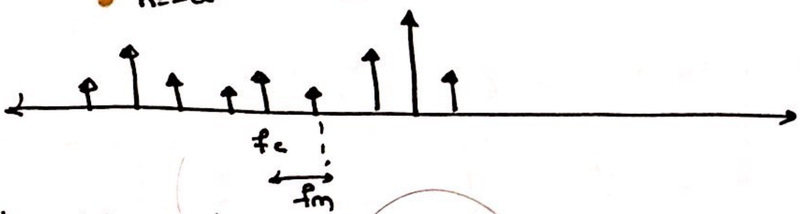
Carson's rule for narrow band FM

An approximate rule for the transmission bandwith of FM

$$B_T \approx 2\Delta f + 2f_m = 2\Delta f\left(1+\frac{2f_m}{2\Delta f}\right) = 2\Delta f\left(1+\frac{1}{\beta}\right)$$

Transmission bandwith of wideband FM

$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)\right]$$



$|J_n(\beta)| > 0.01$

we may define the transmission of an FM wave as the separation between two frequencies beyond which none of the side frequencies of greater than are percent of the carrier.
. amplitude obtained when the modulation is removed

modulation index β	Number of significant side freq. $2n_{max}$
0.1	2
0.5	4
1	6
5	16
10	28

$\boxed{B_T = 2n_{max} f_m}$ //single tone wideboth

where $n_{max} = \max_n |J_n(\beta)|$

Transmission bandwith for arbitrary message signal

. The highest frequency of $m(t)$ is demoted by W

. Deviation ratio : $D = \frac{\Delta f}{W}$

. The deviation ratio plays the role for nonsinusoidal modulation that the modulation index β plays for the case of sinusoidal modulation.

Generilized Corson's rule

$$B_T = 2\Delta f + 2W = 2\Delta f\left(1+\frac{W}{\Delta f}\right) = 2\Delta f\left(1+\frac{1}{D}\right)$$

example: In North America, the max value of freq. deviation Δf is fixed at 75 kHz for commercial FM broadcasting

If we take $W = 15$ kHz

$$D = \frac{\Delta f}{W} = \frac{75.10^3}{15.10^3} = 5$$

$$B_T = 2(\Delta f + W) = 2(75.10^3 + 15.10^3) = 180 \text{ kHz}$$

Generation of FM signals

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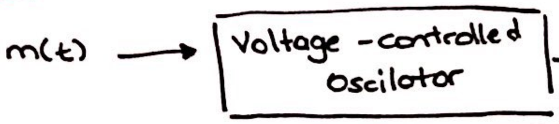
1) Direct Method

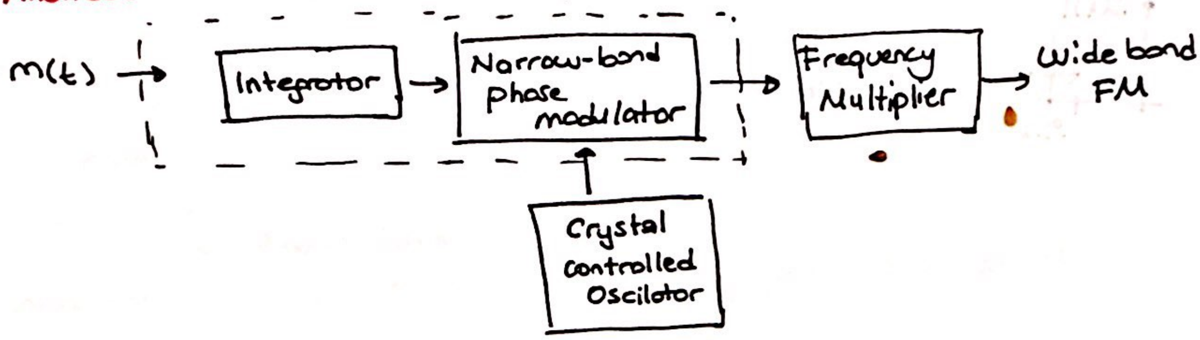


$$s(t) = A_c \cdot \cos\left(2\pi f_c t + k_f \int_0^t m(\tau)\, d\tau\right)$$

Limitations:

- It is capable of providing large frequency deviations
- It has the tendency for the carrier freq. drift.

2) Indirect Method narrowband FM modulator



Frequency Multiplier

$$v(t) = a_1 s(t) + a_2 s^2(t) + \cdots + a_n s^n(t)$$

$$s(t) = \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

For $\underline{n=2}$

$$v(t) = a_1 s(t) + a_2 s^2(t),$$

$$v(t) = a_1 \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right) + a_2 \cos^2\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

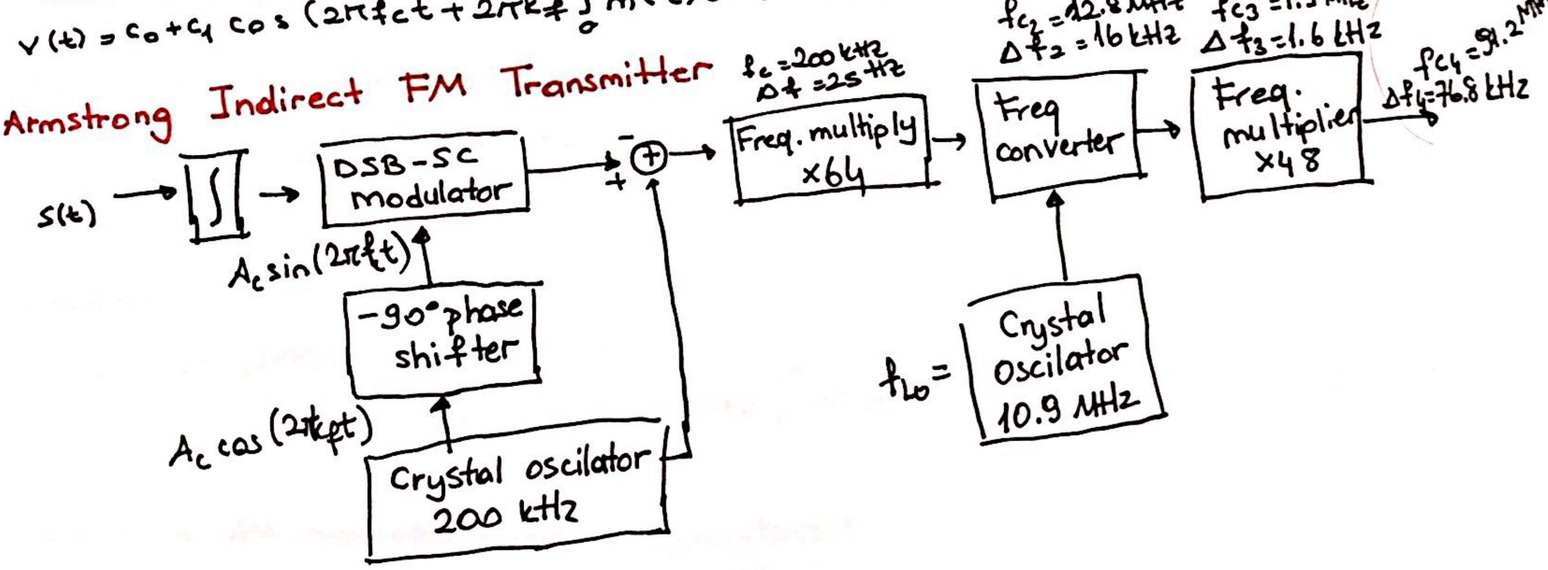
$$v(t) = a_1 \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right) + \frac{a_2}{2} \cos\left(4\pi f_c t + 4\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

$$= \frac{a_2}{2} + a_1 \cos\left(2\pi f_c(t) + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

General Case

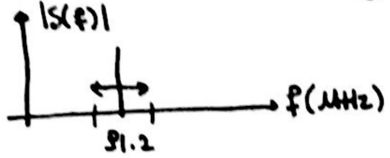
$$v(t) = c_0 + c_1 \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right) + \cdots + c_n \cos\left(n 2\pi f_c t + n 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

Armstrong Indirect FM Transmitter



$f_{c2} + f_{LO} = 23.8$ MHz up converter
$f_{c2} - f_{LO} = 1.9$ MHz down converter

- In order to achieve $\Delta f = 75$ kHz
  We need a multiplication factor of $\frac{75000}{25} = 3000$
  This can be done by using two multipliers, 64 and 48
  Total multiplication factor is $64 \times 48 = 3072$ and $\Delta f = 74.8$ kHz
  Multiplication of $f_c = 200$ kHz by 3072 would field a find carrier about 600 MHz.



Example: An angle modulated signal $s(t)$ with carrier frequency $f_c = 10^5\,Hz$ is described by following equation.

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$$s(t) = 10.\cos(2\pi f_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

mesaj işareti ($= 2\pi k_f$ mesaj işareti)

a. Find the power of modulated signal
b. Find the frequency deviation Δf
c. Find the deviation ratio D
d. Estimate the transmission bandwidth of $s(t)$

a. $P = \frac{A^2}{2} = \frac{10^2}{2} = 50$ watt

b. Freq. deviation
frekans sapması
önce anlık frekansı bulcan

Instanteneous freq. $f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}(2\pi f_c t + 5\sin 3000t + 10\sin 2000\pi t)$

anlık frekans

$$f_i = f_c + \frac{15000}{2\pi}\cos(3000t) + 10000\cos(2000\pi t)$$

$$\Delta f = \max|f_c - f_i| = \max\left|\frac{15000}{2\pi}\cos(3000t) + 10000\cos(2000\pi t)\right| = \frac{15000}{2\pi} + 10000 \approx 12,387\ kHz$$

f_c etrafında bu kadar sapıyor.

c. $D = \frac{\Delta f}{W} \rightarrow$ mesaj işaretinin aldığı max. frekans değer

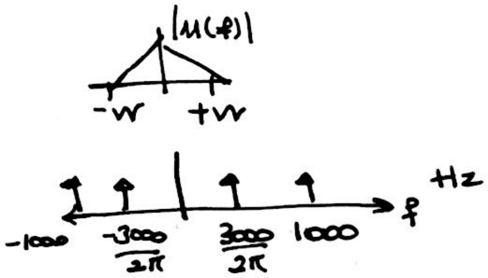
mesaj işareti: $5\sin 3000t + 10\sin 2000\pi t$

bunu almıyoruz — bunun frekansı daha yüksek

W: Highest freq. component of the message signal

$$W = \frac{2000\pi}{2\pi} = 1000\,Hz$$

$$D = \frac{12,387 \times 10^3}{10^3} = 12,387$$



. single tone olsaydı iki tane cos olurdu.
. single tone olsaydı $B_T = 2f_m$ olacaktı.

d. Single tone değil. Genel durumu alcaz.

$B_T = 2(\Delta f + W) \rightarrow$ Genel formül

$= 2(12387 + 1000) = 26774\ Hz$

Demodulation of FM signal

1) Frequency Discriminator

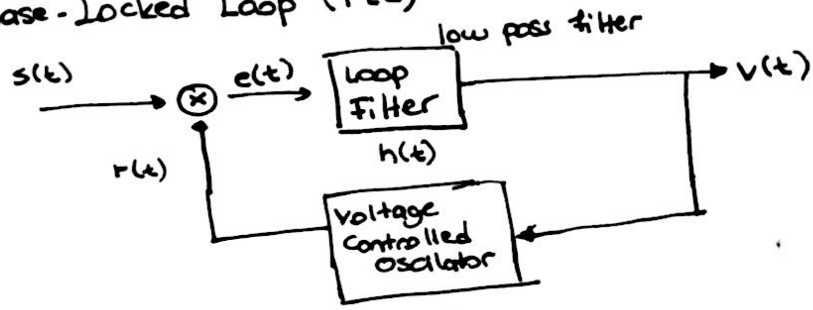
$$s(t) = A_c \cdot \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$$

$$\frac{ds(t)}{dt} = A_c(2\pi f_c + 2\pi k_f m(t))\sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right) = -A_c 2\pi f_c\left(1 + \frac{k_f}{f_c}m(t)\right)\sin(\ldots)$$

. This is an AM modulated signal, if $\frac{k_f \max|m(t)|}{f_c} < 1$

We can use envelope detection to extract the message signal.

2) Phase-Locked Loop (PLL)



$s(t) = A_c \sin(2\pi f_c t + \phi_1(t))$

$\phi_1(t) = 2\pi k_f \int_0^t m(\tau)d\tau$

$r(t) = A_v \cos(2\pi f_c t + \phi_2(t))$

$\phi_2(t) = 2\pi k_v \int_0^t v(\tau)d\tau$

$e(t) = s(t)r(t) = A_c A_v \sin(2\pi f_c t + \phi_1(t)) \cos(2\pi f_c t + \phi_2(t))$

$v(t) = h(t) * e(t) = \frac{A_c \cdot A_v}{2} h(t) * \sin(\cancel{4\pi f_c t} + \phi_1(t) + \phi_2(t))$ //yüksek frekansı geçirmiyor.

$= \frac{A_c \cdot A_v}{2} h(t) * \sin(\phi_2(t)) = \frac{A_c \cdot A_v}{2} h(t) * \sin(\phi_e(t))$

$\phi_e(t) = \phi_1(t) - \phi_2(t)$

$v(t) = \frac{A_c A_v}{2} \int_{-\infty}^{+\infty} h(t-x) \sin(\phi_e(x))dx$

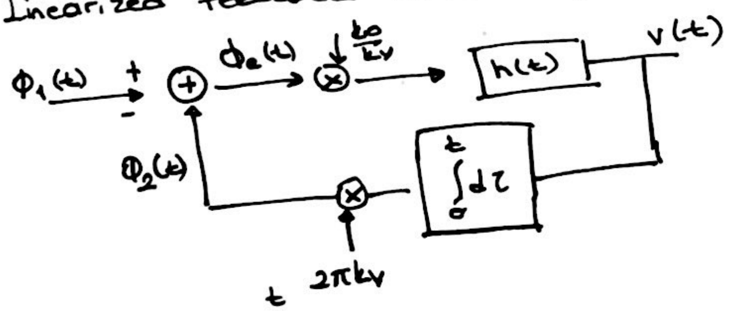
For small $\phi_e(x)$ $\qquad \sin(\phi_e(x)) \approx \phi_e(x)$ near-phase-lock

$v(t) \simeq \frac{A_c A_v}{2} \int h(t-x)\phi_e(x)dx$ $\qquad K_o = \frac{k_v A_c A_v}{2}$ loop gain

$\simeq \frac{k_o}{k_v} \int h(t-x)\phi_e(x)dx$

For $\phi_e(t) = \phi_1(t) - \phi_2(t) = 0$ phase-lock

Linearized feedback model (doğrulaştırılmış geri besleme modeli)



Amaç: faz farkını sıfıra götürmek

$\phi_2(t) = 2\pi k_v \int_0^t v(\tau)d\tau$

$\phi_2'(t) = 2\pi k_v v(t) \rightarrow v(t) = \frac{1}{2\pi k_v}\phi_2'(t)$

$\phi_2(t) = \phi_1(t) - \phi_e(t) = 2\pi k_f \int_0^t m(\tau)d\tau - \phi_e(t)$

For small: $\phi_e(t)$ $\quad v(t) \approx \frac{k_f}{k_v} m(t)$

Stereo FM / sağ ve sol hoparlerden farklı sesler gelebiliyor.

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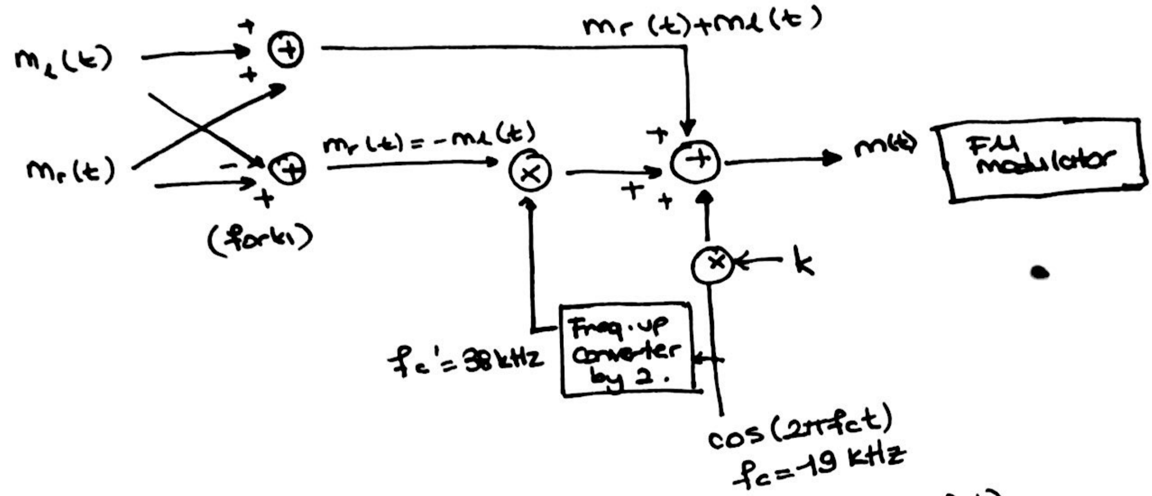
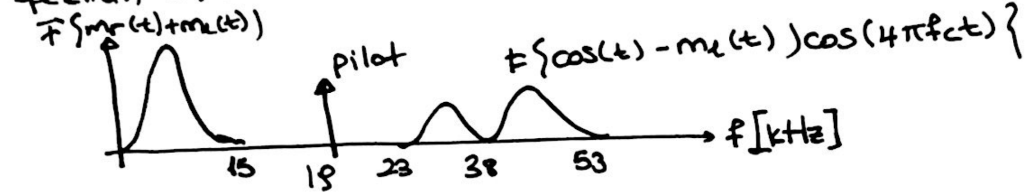
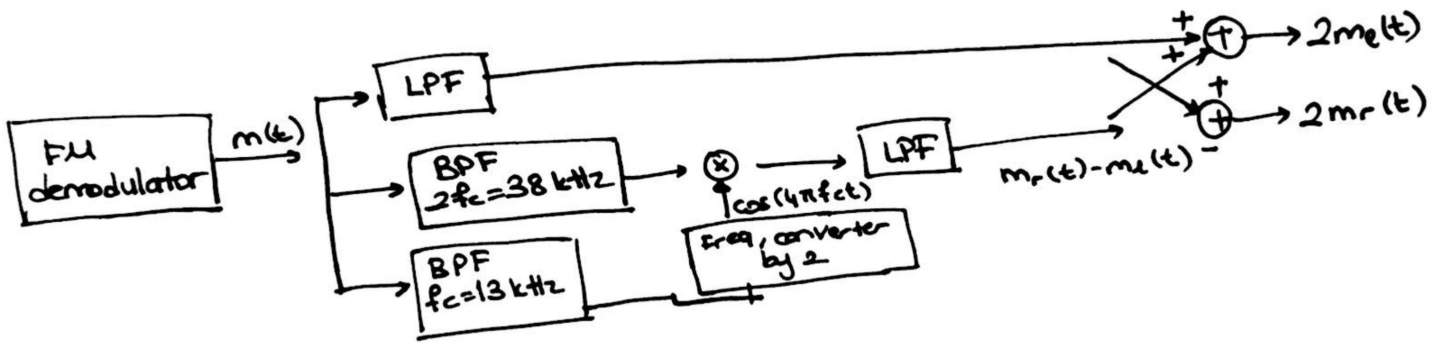
sol mikrofondan kayıt yaptığını: $m_l(t)$
sağ mikrofondan " " : $m_r(t)$



$$m(t) = [m_r(t) + m_l(t)] + [m_r(t) - m_l(t)]\cos(4\pi f_c t) + k\cos(2\pi f_c t)$$

Spektrum of stereo FM





Probability

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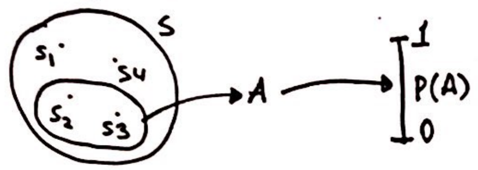
Probability triplet (S, F, P)



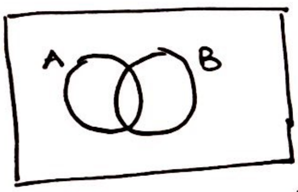
1. Sample space S of elementary outcomes
2. A set F of events that are subsets of S
3. A probability measure $P(A)$ assigned to each event A in the set F $\quad A \in F$

Axioms of Probability

1. $P(A) \geq 0$ for all $A \in F$
2. $P(S) = 1$
3. If $A_1, A_2, \ldots$ are disjoint (or mutually exclusive) events, i.e. $A_i \cap A_j = \emptyset$ the empty set for all $i \neq j$, then $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

Conditional probability and Independence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A,B)}{P(B)}$$

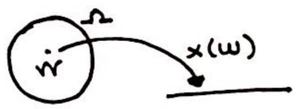


$$P(B \backslash A) = \frac{P(A,B)}{P(A)}$$

Two events are said to be statistically independent iff

$P(A,B) = P(A) \cdot P(B)$ // bağımsızsa (A'nın olması B'yi etkilemiyorsa)

Random Variables



- A random variable is a function from sample space Ω to real line

$x(w): \Omega \rightarrow \mathbb{R}$

- If $X \in \mathbb{R}$, random variable is said to be continuous. It is completely specified by its probability density function (pdf) $f_X(x)$ or its cummulative distribution function (cdf)
- If $X \in \mathbb{Z}$, integer random variable is said to be discrete. It is completely specified by its probability mass function (pmf) $P_X(x)$ or its cdf $F_X(x)$

Cummulative Distribution Function (cdf)

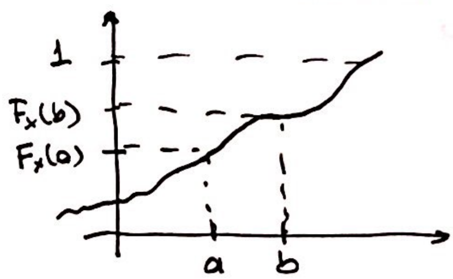
$F_X(x) = P(X \leq x)$

$F_X(a) = P(X \leq a)$

Properties

1) $F_X(x) \geq 0$ is monotocially nondecreasing.

If $b > a$, then $F_X(b) \geq F_X(a)$

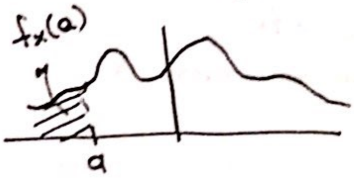


$P(a < X \leq b) = F_X(b) - F_X(a)$

2) $\lim_{x \to \infty} F_X(x) = 1 \qquad \lim_{x \to -\infty} F_X(x) = 0$

Probability Density Function (pdf)

$$F_x(x) = \int_{-\infty}^{x} f_x(t)\,dt$$



$$f_x(x) = \frac{dF_x(x)}{dx}$$

Properties:

1. $f_x(x) \geq 0$
2. $\int_{-\infty}^{+\infty} f_x(x)\,dx = 1$
3. $P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} f_x(x)\,dx$

Expectations:

1. Mean (or first moment)

$$E[x] = \int_{-\infty}^{+\infty} x\, f_x(x)\,dx$$

2. Second moment (or quorage power)

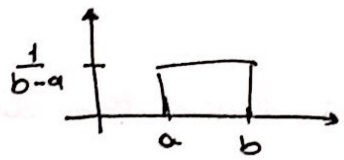
$$E[x^2] = \int_{-\infty}^{+\infty} x^2 \cdot f_x(x)\,dx$$

3. $Var(x) = E\left[(x - E[x])^2\right] = E[x^2] - (E[x])^2$

Uniform random variables

$X \sim U[a,b]$ for $b > a$

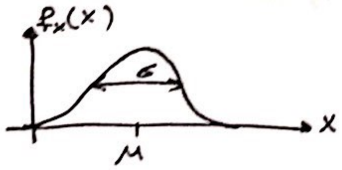
$$f_x(x) = \begin{cases} \frac{1}{b-a}, & \text{for } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$



Gaussian (Normal) random variable

$X \sim N(\mu, \sigma^2)$

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$E[x] = \mu \rightarrow$ beklenendeğer

$Var(x) = \sigma^2$

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Two random variables

Joint pdf $f_{x,y}(x,y)$

$$P((x,y)\in A) = \iint_{(x,y)\in A} f_{x,y}(x,y)\,dx\,dy$$

for a rectangular area

$$P(a<x\leq b,\ c<y\leq d) = \int_c^d \int_a^b f_{x,y}(x,y)\,dx\,dy$$

Properties

1) $f_{x,y}(x,y)\geq 0$

2) $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{x,y}(x,y)\,dx\,dy = 1$

Marginal pdf

$$f_x(x) = \int_{-\infty}^{+\infty} f_{x,y}(x,y)\,dy$$

Conditional pdf

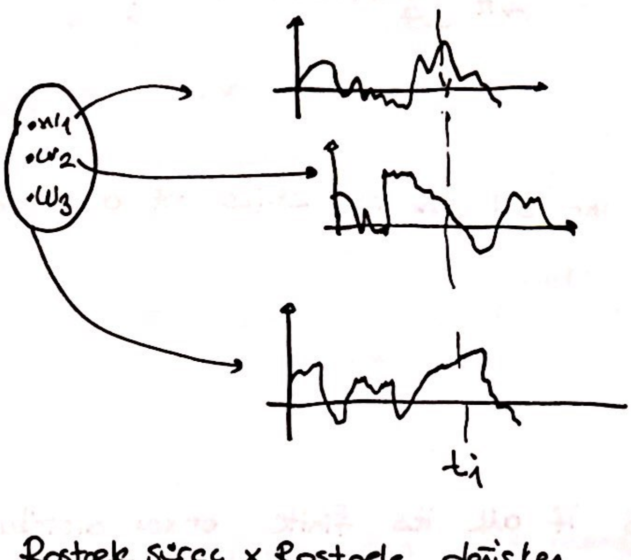
$$f_{y|x}(y/x) = \frac{f_{x,y}(x,y)}{f_x(x)} \quad \text{for } f_x(x)\neq 0$$

Independence: Two conditinuous random variable are said to be independent if

$f_{x,y}(x,y) = f_x(x)\cdot f_y(y)$ for all x,y

Stocheistic processes

A stochastic (also called random) process $x(t)$ is an infinite collection of random variables, one for each value of time $(t\in\tau)$



Zaman sabitlersen bu rastgele değişken olmuş olur.

t_1 anında 3 farklı değer alabilir.

Rastgele süreç x Rastgele değişken arasındaki fark bu.

For $t = t_1$ fixed

$X(t_1, \omega)$ is a random variable

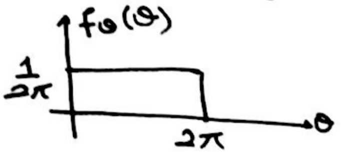
Continuous-time stochastic processes

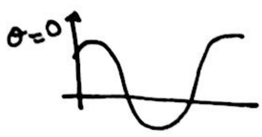
- A random process is continuous if T is uncountably infinite

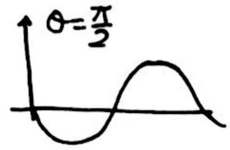
Example: Sinusoidal signal with random phase

$$X(t) = \alpha \cos(\omega t + \theta)$$

$\theta \sim U[0, 2\pi]$







Auto-correlation function of a stochastic process

- For a random process $X(t)$ the first and second order moments are specified by
- mean function $\mu_X(t) = E[X(t)]$
- auto correlation function: $R_X(t_1, t_2) = E[X(t_1), X(t_2)]$
- the auto covariance function

$$C_X(t_1, t_2) = E\left[(X(t_1) - E[X(t_1)])\right](X(t_2) - E[X(t_2)]) = R_X(t_1, t_2) - \mu_X(t_1)\mu_X(t_2)$$

example: Random phase signal

$$X(t) = \alpha \cos(\omega t + \theta) \qquad \theta \sim U[0, 2\pi]$$

$$\mu_X(t) = E[X(t)] = E[\alpha \cos(\omega t + \theta)] = \int_{-\infty}^{+\infty} \alpha \cos(\omega t + \theta) f_\theta(\theta) d\theta = \int_0^{2\pi} \alpha \cos(\omega t + \theta) \frac{1}{2\pi} d\theta$$

$$= \frac{\alpha}{2\pi} \sin(\omega t + \theta) \Big|_0^{2\pi} = 0$$

$$R_X(t_1, t_2) = E[X(t_1), X(t_2)] = E[\alpha^2 \cos(\omega t_1 + \theta) \cos(\omega t_2 + \theta)] = \int_{-\infty}^{+\infty} \alpha^2 \cos(\omega t_1 + \theta) \cos(\omega t_2 + \theta) f_\theta(\theta) d\theta$$

$$= \frac{\alpha^2}{4\pi} \int_0^{2\pi} \cos(\omega(t_1 + t_2) + 2\theta) + \cos(\omega(t_1 - t_2)) d\theta = \frac{\alpha^2}{4\pi} \left[\frac{1}{2} \sin(\omega(t_1 + t_2) + 2\theta)\Big|_0^{2\pi} + \theta \cos(\omega(t_1 - t_2)\right.$$

$$R_X(t_1, t_2) = \frac{\alpha^2}{2} \cos(\omega(t_1 - t_2))$$

Stationarity

- Stationarity refers to time invariance of some or all the statistics of a random process.
- We define two types of stationarity (SSS)

  strict sense stationary (SSS)

  wide " " (WSS)

- A random process $X(t)$ is said to be SSS if all its finite order distributions are time invariant.

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Wide Sense Stationary (WSS) Random Process

- A random process $X(t)$ is said to be WSS if its mean and autocorrelation functions are time invariant.

$$E[x(t)] = \mu$$

$$R_x(t_1,t_2) = R_x(t_1-t_2) = R_x(\tau)$$

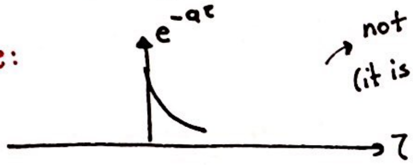
Autocorrelation function of WSS process

1. $R_x(\tau)$ is real and even $R_x(\tau) = R_x(-\tau)$
2. $R_x(\tau) \le R_x(0) = E[x^2(t)]$ the average power of $X(t)$
3. If $R_x(T) = R_x(0)$ for some T, when $R_x(\tau)$ is periodic with period T and so is $X(t)$

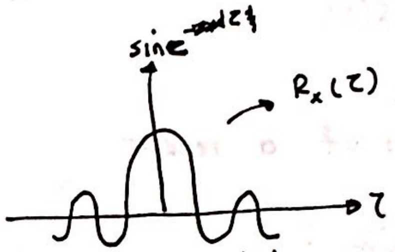
Example: $x(t) = a.\cos(\omega t + \theta)$ random phase

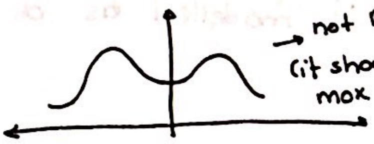
$$R_x(\tau) = \frac{a^2}{2}\cos(\omega \tau)$$

Example:

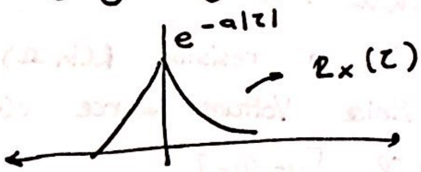


not $R_x(\tau)$ (it is not symmetrical)

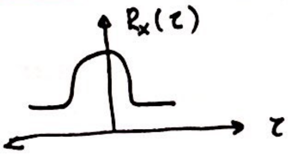


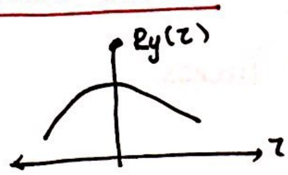


not $R_x(\tau)$ (it should have max value at origin)



Interpretation Of Autocorrelation Function





If $R_x(\tau)$ drops quickly with τ, this means that samples become uncorrelated quickly we increase τ.

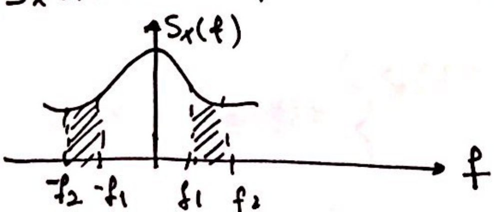
Power Spectral Density (PSD) of a WSS Process

$$S_x(f) = \mathcal{F}\{R_x(\tau)\} = \int_{-\infty}^{+\infty} R_x(\tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$R_x(\tau) = \mathcal{F}^{-1}\{S_x(f)\} = \int_{-\infty}^{+\infty} S_x(f).e^{j2\pi f \tau}\, df$$

Properties Of PSD

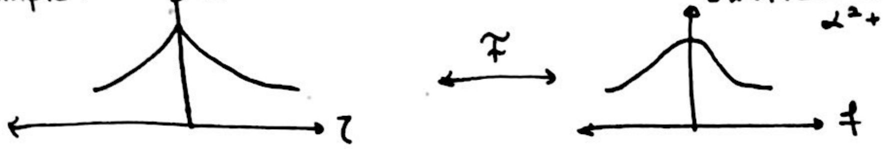
1. $S_x(f)$ is real and even.
2. $S_x(f)$ is the power density. The average power of $x(t)$ in frequency band $[f_1, f_2]$ is



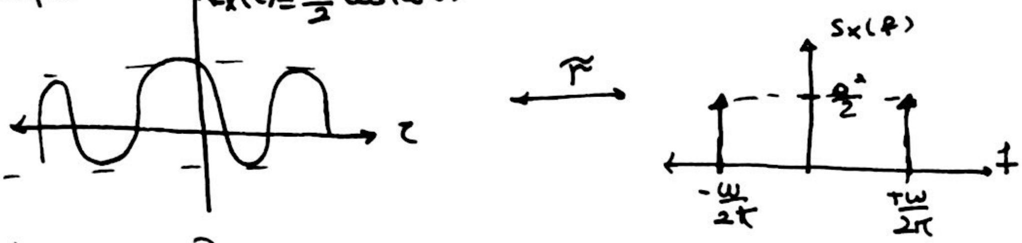
The avarage power of $x(t)$

$$E[x^2(t)] = \int_{-\infty}^{+\infty} S_x(f)\, df$$

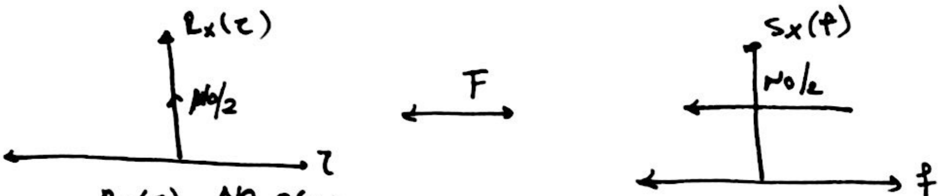
Example: $R_x(\tau) = e^{-2\alpha|\tau|}$ $\quad S_x(f) = \frac{\alpha}{\alpha^2 + (sf)^2}$



Example: $R_x(\tau) = \frac{a^2}{2}\cos(\omega\tau)$



White Noise Process



$R_x(\tau) = \frac{N_0}{2}\delta(\tau)$ $\qquad S_x(f) = \frac{N_0}{2}$ for all f

Thermal Noise:

Noise equivalent of a resistor

The noise in a resistor R(in Ω) due to thermal noise is modelled as a white Gaussian Noise Voltage source v(t) with psd.

$S_x(f) = 2kTR \quad [V^2/Hz]$

k: Boltzman constant

T: Temperature [Kelvin]

Response of LTI System to DUSS Process

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

LTI

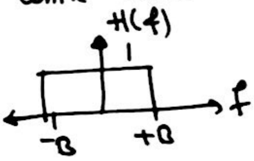
$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau$$

$$S_y(f) = |H(f)|^2 \cdot S_x(f)$$

Ideal Low-Pass Filtered White Noise.

$w(t) \longrightarrow \boxed{h(t)}$

white noise with psd $\frac{N_0}{2}$ for all f



a) $S_N(f) = |H(f)|^2 S_{(x)}(f) = \begin{cases} \frac{N_0}{2}, & -B < f < B \\ 0, & \text{otherwise} \end{cases}$

$$S_N(f) = \frac{N_0}{2}\,\text{rect}\left(\frac{f}{2B}\right)$$

b) Average power of $N(t)$

$$E[N^2(t)] = \int_{-\infty}^{+\infty} S_N(f)\,df = \int_{-B}^{+B} \frac{N_0}{2}\,df = N_0 B$$

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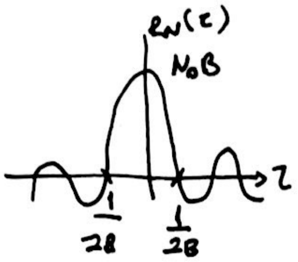
c) $R_N(\tau) = \int_{-\infty}^{+\infty} S_N(f) e^{j2\pi f\tau} df = \int_{-B}^{+B} \frac{N_0}{2} e^{j2\pi f\tau} df = \frac{N_0}{2} \frac{1}{j2\pi\tau} e^{j2\pi f\tau} \Big|_{-B}^{+B}$

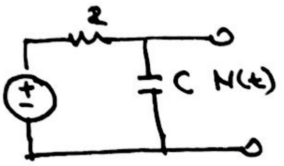
$= \frac{N_0}{2} \frac{\sin(2\pi B\tau)}{\pi\tau} = N_0 B \,\mathrm{sinc}(2B\tau)$

$\xleftrightarrow{F^{-1}}$



(Average power of $N(t)$
$E[N^2(t)] = R_N(0) = N_0B$)

RC Low-Pass Filtered White Noise



$H(f) = \frac{1}{1+j2\pi fRC}$

$|H(f)|^2 = \frac{1}{1+(2\pi fRC)^2}$

Output PSD $S_N(f) = |H(f)|^2 S_W(f) = \frac{2kTR}{1+(2\pi fRC)^2}$

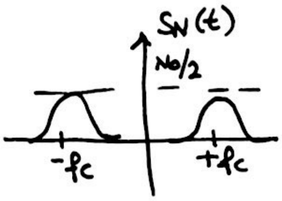
The average output power

$E[N^2(t)] = \int_{-\infty}^{+\infty} S_N(f) df = 2kTR \int_{-\infty}^{+\infty} \frac{1}{1+(2\pi fRC)^2} df = \frac{kT}{\pi C} \int_{-\infty}^{+\infty} \frac{1}{1+x^2} dx = \frac{kT}{\pi C} \arctan(x) \Big|_{-\infty}^{+\infty} = \frac{kT}{C}$

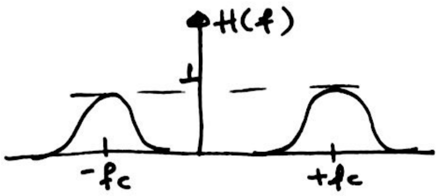
$x = 2\pi fRC$
$dx = 2\pi RC\,df$

Narrow Band Noise

$w(t) \rightarrow \underset{\text{BPF}}{\boxed{H(f)}} \rightarrow N(t)$



white noise process with psd $\frac{N_0}{2}$



$S_N(f) = |H(f)|^2 S_W(f)$

$N(t)$ is called narrow-band noise

$N(t) = N_I(t)\cos(\omega_c t) - N_Q(t)\sin(\omega_c t)$

$N_I(t)$ = Inphase component of $N(t)$

$N_Q(t)$ = quadrature

Properties of $N(t)$

1) $N_I(t)$ and $N_Q(t)$ of narrow band noise $N(t)$ have zero mean

2) If $N(t)$ is Gaussian, then $N_I(t)$ and $N_Q(t)$ are Gaussian

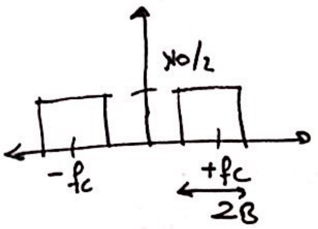
3) If $N(t)$ is stationary, then $N_I(t)$ and $N_Q(t)$ are stationary.

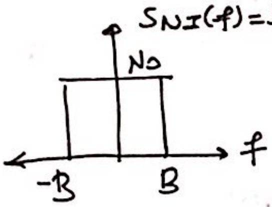
4) Both $N_I(t)$ and $N_Q(t)$ have the same psd. This psd is related to the psd of $N(t)$ by

$$S_{N_I}(f) = S_{N_Q}(f) = \begin{cases} S_N(f-f_c) + S_N(f+f_c), & -B \le f \le B \\ 0, & \text{otherwise} \end{cases}$$

5. $N_I(t)$ and $N_Q(t)$ have the same variance as $N(t)$

$S_{N_I}(f) = S_{N_Q}(f)$





Average power of $N(t)$ $\quad E[N^2(t)] = \int_{-\infty}^{+\infty} S_N(f)\,df = 2 \cdot N_o \cdot B$

Average power of $N_I(t)$ $\quad E[N_I^2(t)] = 2N_oB$

Noise in Analog Communications

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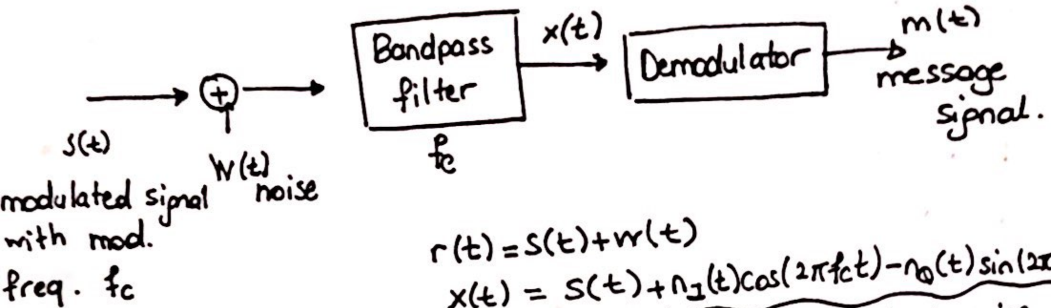


$r(t) = s(t) + w(t)$

$x(t) = s(t) + \underbrace{n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)}_{\text{narrow band noise } n(t)}$

Signal to Noise Ratio (SNR)

$$SNR = \frac{\text{avarage power of the signal}}{\text{avarage power of the noise}} = \frac{E[s^2(t)]}{E[n^2(t)]}$$

Sinusoidal signal in noise:

$r(t) = A_c \cos(2\pi f_c t) + w(t)$ → • w(t) zero mean with psd $\frac{N_0}{2}$ white nose. [W/Hz]

$x(t) = A_c \cdot \cos(2\pi f_c t) + n(t)$

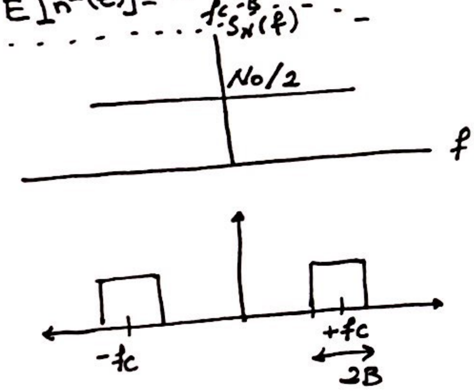
Signal Power:

• $E[s^2(t)] = E[A_c^2 \cos^2(2\pi f_c t)]$ by using time avarage $= \frac{1}{T}\int_0^T A_c^2 \cos^2(2\pi f_c t)\,dt$

$= \frac{A_c^2}{2T}\int_0^T (1+\cos(4\pi f_c t))\,dt = \frac{A_c^2}{2T}\left[t + \frac{\sin(4\pi f_c t)}{4\pi f_c}\right]\Big|_0^T = \frac{A_c^2}{2}\,W$

Noise Power:

$E[n^2(t)] = 2 \cdot \int_{f_c-B}^{f_c+B} \frac{N_0}{2}\,df$

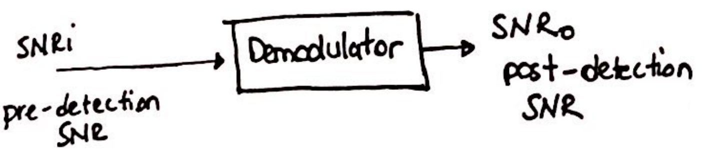


$$E[n^2(t)] = \int_{-\infty}^{+\infty} S_N(f)\,df = 2 \cdot \int_{f_c-B}^{f_c+B} \frac{N_0}{2}\,df = 2N_0B\ W$$

$$SNR = \frac{\frac{A_c^2}{2}}{2N_0B} = \frac{A_c^2}{4N_0B}$$

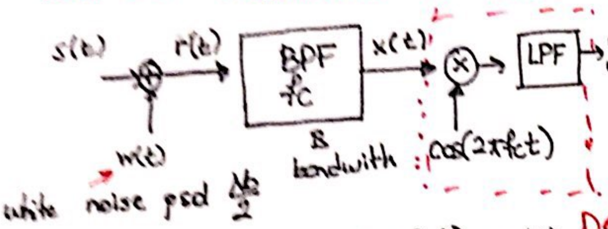
$SNR_{dB} = 10\log_{10} SNR\ [dB]$ gürültü büyükse SNR negatif çıkar.

Figure of merit



$$\text{Figure of merit} = \frac{SNR_o}{SNR_i}$$

DSB-SC modulation in noise



$s(t)=A_c.m(t)\cos(2\pi f_c t)$ DSB-SC modulated signal.

$r(t)=s(t)+w(t)$ received signal.

$x(t)=A_c.m(t).\cos(2\pi f_c t)+n(t)$

↳ narrowband noise

Pre-detection SNR

$$E[s^2(t)] = E[m^2(t)A_c.\cos(2\pi f_c t)^2] = E[m^2(t)].E[A_c.\cos(2\pi f_c t))^2]$$

$$= P\frac{A_c^2}{2}$$

$$E[n^2(t)] = 2N_0B$$

$$SNR_i = \frac{E[s^2(t)]}{E[n^2(t)]} = \frac{A_c^2}{4N_0B}$$

girişteki SNR

Output of the mixer

$$v(t)=x(t).\cos(2\pi f_c t) = \Big[[A_c.m(t).\cos(2\pi f_c t)] + \underbrace{n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin 2\pi f_c t}_{n(t)}\Big].\cos(2\pi f_c t)$$

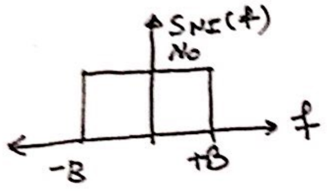
$$= \frac{1}{2}\left[A_c.m(t)+n_I(t)\right] + \frac{1}{2}\Big[[A_c.m(t)+n_I(t)]\cos(4\pi f_c t)\Big] - \frac{1}{2}n_Q(t)\sin(4\pi f_c t)$$

Output of lowpass filter

$$y(t) = \underbrace{\frac{1}{2}A_c m(t)}_{\text{signal}} + \underbrace{\frac{1}{2}n_I(t)}_{\text{noise}}$$

$$E[s_o^2(t)] = E[(A_c.m(t))^2] = A_c^2.P$$

$$E[n_I^2(t)] = \int_{-\infty}^{+\infty} S_{NI}(f)df = \int_{-B}^{+B} N_0.df = 2N_0B$$



Pre detection SNR : $SNR_o = \frac{\frac{1}{4}.A_c^2.P}{\frac{1}{4}.2N_0B} = \frac{P.A_c^2}{2N_0B}$

Figure of merit $= \frac{SNR_o}{SNR_i} = 2$? → yarınki derste kontrol edilecek. Hoca notlarında 1 bulmuş.