Kaynağı — Bilgisi

Verici (Transmitter) : Mesaj iletimini kanal üzerinden iletilecek hale getirir.

Kanal (Channel) : Alıcı ile vericiyi birleştiren fiziksel ortamdır. Kablo, hava vb.

Alıcı (Receiver) : Gelen işareti aslına uygun bir biçimde geri elde eder.

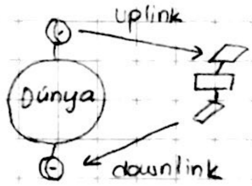
Ders Kitapları:

- S. HAYKIN, M. MOHER INTRODUCTION TO ANALOG AND DIGITAL COMMUNICATION, WILEY, 2
- B. P. LATHI, DIGITAL AND ANALOG COMMUNICA SYSTEM, OXFORD, 1998
- Ahmet Hamdi KARAYAN, ANALOG HABERLEŞME, BİRSEN

Haberleşme sistemi örneği

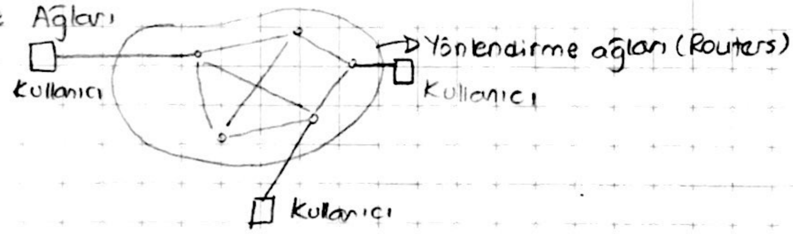
Radyo, TV yayıncılığı tek yönlüdür. Vericiden alıcıya doğrudur.

Uydu haberleşmesi

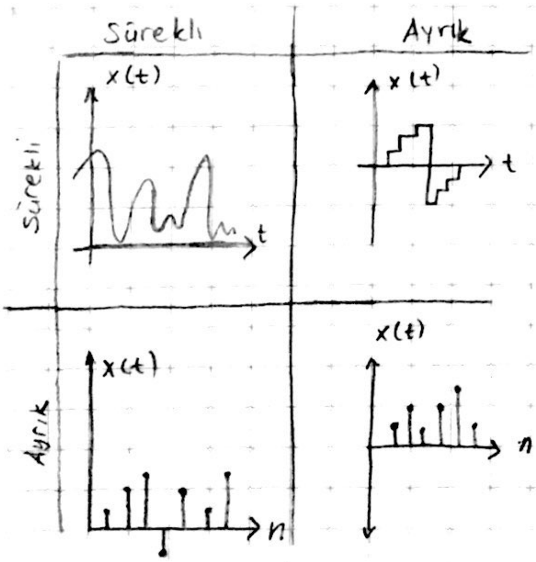


Çift yönlü kullanılabilir.

Haberleşme Ağları

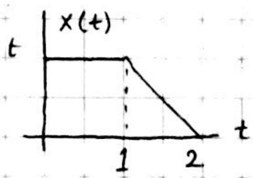


İŞARETLER



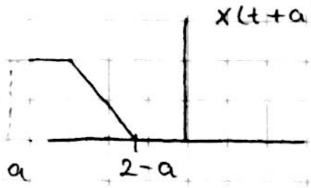
herhangi bir n değeri alınabilir. — sadece belli n değerleri alınabilir

İŞARETLERDE İŞLEMLER



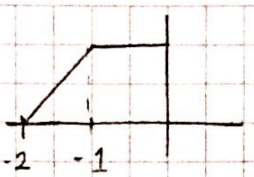
zamanda öteleme (time shift)

$x(t+a) \Rightarrow a>0$



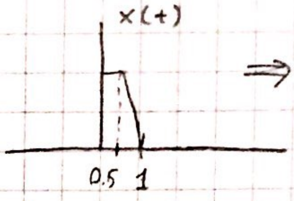
zamanda ters çevirme

// time inversing

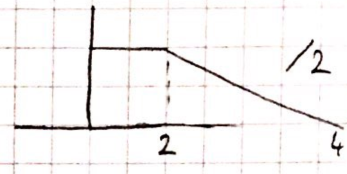


zamanda ölçekleme

// time scaling



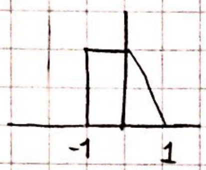
$\Rightarrow \times 2$



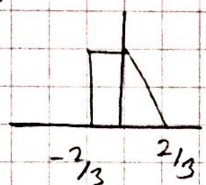
$/2$

$\Rightarrow x\left(\frac{3t}{2}+1\right)$

1. YOL; ÖNCE ÖTELEME;

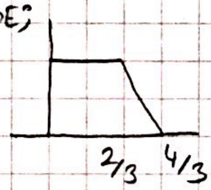


SONRA ÖLÇEKLEME;

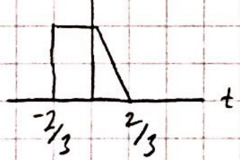


2. YOL; ÖNCE ÖLÇEKLEME;

$x\left(\frac{3}{2}\left(t+\underbrace{\frac{2}{3}}_{\text{öteleme miktarı}}\right)\right)$

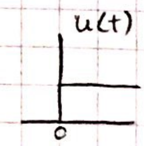


ÖTELEME;

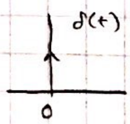


BAZI ÖZEL İŞARETLER

Birim basamak fonksiyonu (unit step function) $u(t)=\begin{cases}0 & t<0\\ 1 & t\geq 0\end{cases}$

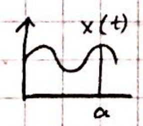


Birim dürtü fonksiyonu (unit impulse function) $\delta(t)=\frac{du(t)}{dt}$

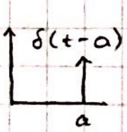


$$\int_{-\infty}^{\infty}\delta(t).dt=1$$

$\int_{-\infty}^{\infty}x(t).\delta(t-a).dt = x(a)$

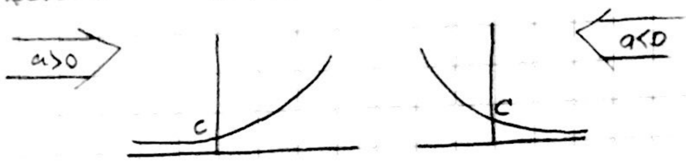


$\times$



$= x(a)$

üstel işaretler $x(t) = c.e^{at}$

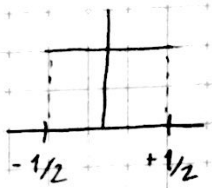


Dikdörtgen işaret // Rectangular signal // Karesel işaret

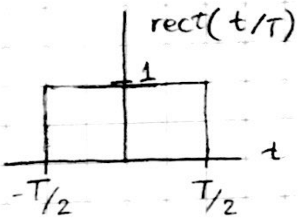
$$\text{rec}(t) = \Pi(t) = \begin{cases} 0, & |t| > 1/2 \\ 1/2, & t = 1/2 \\ 1, & |t| < 1/2 \end{cases}$$

?

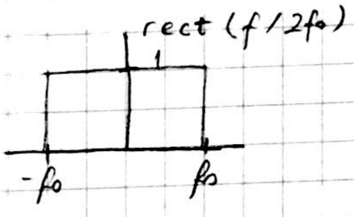
$\Rightarrow$



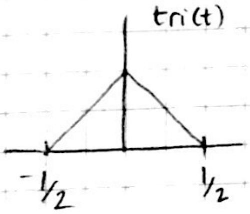
Ölçeklenmiş karesel işaret



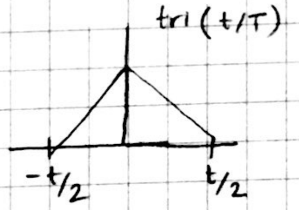
or



Üçgensel işaret //Triangular signal (Birim //Unit)

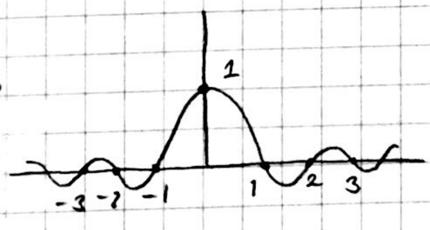


$\Rightarrow$ ölçeklenmiş



Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t} \Rightarrow$$



$\Rightarrow$ Kare dalganın fourier dönüşümü alındığında sinc fonksiyonu elde edilir

Sinusoidal function

$x(t) = A.\cos(w_0 t + \Phi) \Rightarrow$

$\Phi : 90°$

A : Amplitude

w_0 : Angular frequency

$w_0 = \dfrac{2\pi}{T_0}$ rad/s

Φ : phase

Complex Exponentials

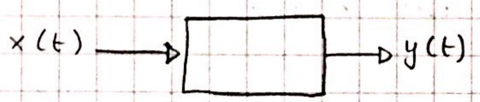
Euler formula: $e^{\pm j w_0 t} = \cos w_0 t \pm j \cdot \sin w_0 t \qquad j = \sqrt{-1}$

$$\cos w_0 t = \frac{e^{j w_0 t} + e^{-j w_0 t}}{2}$$

$$\sin w_0 t = \frac{e^{j w_0 t} - e^{-j w_0 t}}{2j}$$

Systems

Properties of Continuous - time systems :



1) Hafızalı sistemler : Geçmişteki değerleri kullanır
2) Nedensel sistemler : şimdiki ve geçmişteki değerlere bağlı

3) Time - invariance ; zamanda değişmeme

$x(t) \to y(t) \implies x(t+a) \to y(t+a)$

Example:

$$y(t) = \sin(x(t))$$

$x_1(t) = x(t - t_0) \to y_1(t) = y(t - t_0) \Rightarrow y(t - t_0) = \sin(x(t - t_0))$

$y_1(t) = \sin(x_1(t))$

Example:

$y(t) = x(2t) \qquad x_1(t) = x(t - t_0) \implies y_1(t) = x_1(2t) = x(2t - t_0)$

$y(t - t_0) = x(2t - 2t_0)$

$y_1(t) \neq y(t)$ so: not time invariance

4) Linearity

= scaling

IF THERE ARE THESE 2 PROPERTY IN A SYSTEM WE CAN CALL THAT SYSTEM AS A LINEAR SYSTEM

= addivity

Example : $y(t) = t.x(t)$ Linear ?

$y_1(t) = a_1 x_1(t) t + a_2 t x_2(t) \rightarrow a_1 y_1(t) + a_2 . y(t)$ ✓

Example:

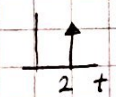
$y(t) = x^2(t) \Rightarrow a_1 x_1(t) + a_2 x_2(t) \rightarrow (a_1 x_1(t) + a_2 x_2(t))^2$ × non linear

LINEAR , TIME INVARIANT SYSTEM (LTI)

$\delta(t) \rightarrow$ [LTI] $\rightarrow h(t)$ unit impulse response of the system

Unit impulse response of an LTI system determines the complete characterization of the system.

$\delta(t) \rightarrow$ [$h(t)$] LTI $\rightarrow h(t)$



$\delta(t-\tau) \rightarrow h(t-\tau)$

time invariant olduğu için çıkış da ötelendi.

$a.\delta(t-\tau) \rightarrow a.h(t-\tau)$ } Linear olduğu için çıkışta çarpıldı.

$$x(t) = \int_{-\infty}^{\infty} x(\tau).\delta(t-\tau).d\tau \rightarrow \boxed{y(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau).d\tau}$$

Konvolüsyon integrali

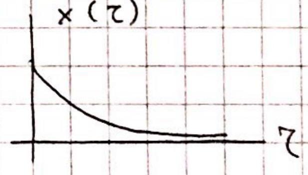
LTI olduğu için

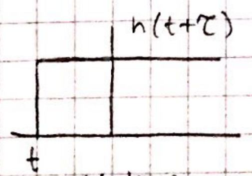
Example:

$x(t) = e^{-at} \rightarrow$ [$h(t) = u(t)$] LTI $\rightarrow y(t) = ?$

$a > 0$

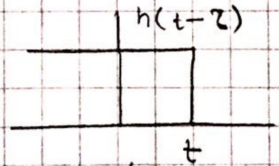
$\rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau).d\tau \Rightarrow$





öteleme

$\Rightarrow$



zamanda ters çevirme

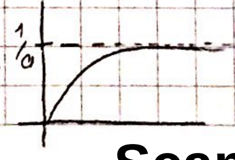
$\Rightarrow$ $t < 0$ $y(t) = 0$

$t > 0$ $y(t) = \int_0^t x(\tau).h(t-\tau).d\tau$

$= \int_0^t e^{-a\tau} . 1 . d\tau$

(*)

$y(t) = \frac{1}{a}(1 - e^{-at}).u(t) \Rightarrow$



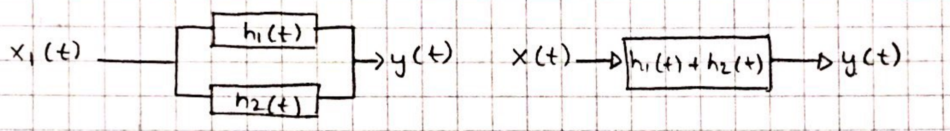
$\frac{e^{-a\tau}}{-a}\Big|_0^t = \frac{e^{-at}}{-a} + \frac{1}{a}$ (*)

Convolution properties

$\Rightarrow$ $x(t) * h(t) = h(t) * x(t)$

$$\Rightarrow \int_{-\infty}^{\infty} x(t).h(t-\tau).d\tau = \int_{-\infty}^{\infty} h(t).x(t-\tau).d\tau$$

$\Rightarrow$ Distributive: $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$



$\Rightarrow$ Associative:

$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$

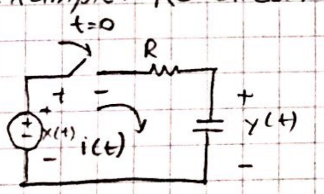
$$x(t) \to \boxed{h_1(t)} \to \boxed{h_2(t)} \to y(t)$$

$$x(t) \to \boxed{h_1(t) * h_2(t)} \to y(t)$$

Casual LTI systems described by linear constant coefficient differential equations.

$$\sum_{k=0}^{N} a_k . \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{N} b_k . \frac{d^k x(t)}{dt^k}$$

Example: RC circuit $x(t) \to \underset{LTI}{\boxed{}} \to y(t)$



$I_0 = 0$ $\quad x(t) = R.i(t) + y(t)$

$$i(t) = C.\frac{dy(t)}{dt}$$

$$x(t) = R.C.\frac{dy(t)}{dt} + y(t)$$

EIGENFUCTIONS OF LTI SYSTEMS

$$e^{j\omega_0 t} \to \underset{LTI}{\boxed{h(t)}} \to \underbrace{H(\omega_0)}_{\text{katsayı ile genliği büyütülüyor}}\overset{\text{eigen value}}{} \overset{\text{eigen function}}{\underline{e^{j\omega_0 t}}}$$

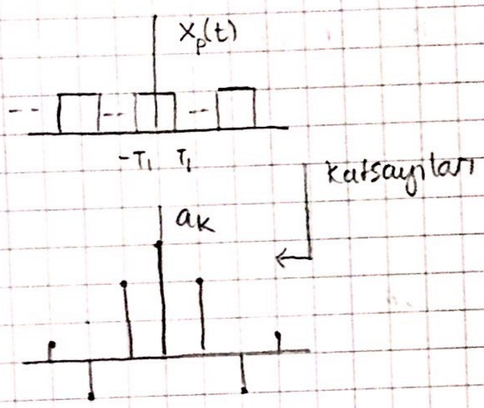
Complex exponentials are the eigenfuctions of LTI systems.

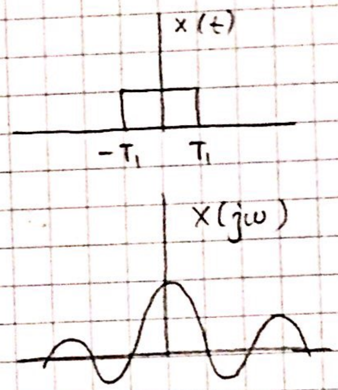
$$y(t) = \int_{-\infty}^{\infty} h(\tau).x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau).e^{j\omega_0(t-\tau)}.d\tau = \underbrace{e^{j\omega_0 t}\int_{-\infty}^{\infty} h(\tau).e^{-j\omega_0 \tau}.d\tau}_{\text{FOURIER TRANSFORM}}$$

$$= e^{j\omega_0 t} \int_{-\infty}^{\infty} \underbrace{h(\tau) . e^{-j\omega_0 \tau} . d\tau}_{\text{Fourier transform of } h(t) \text{ evaulated at } \omega_0} = e^{j\omega_0 t} H(\omega_0)$$

FOURIER SERIES AND FOURIER TRANSFORM ; frekans domaininde analiz için!

Fourier series	Fourier Transform
- Freq. analysis of continuous time <u>periodic</u> signal	- Freq. analysis of continuous time <u>aperiodic</u> signals
- Freq. spectrum is discrete	- Freq. spectrum is continuous





FOURIER SERIES: $x(t) = \sum_{k=-\infty}^{\infty} a_k . e^{-jk\omega_0 t}$

$= \sum_{k=-\infty}^{\infty} a_k . e^{-jk.2\pi.\frac{t}{T_0}}$

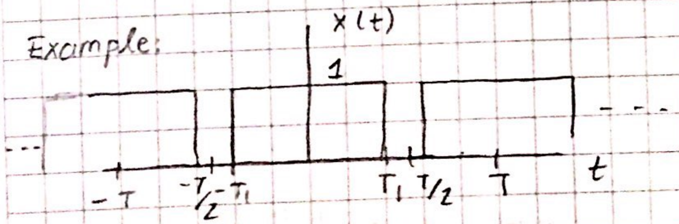
ω_0 : Fundamental angular freq.

T_0 : Fundamental period

$\omega_0 = \frac{2\pi}{T_0}$

$$a_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) . e^{-jk\omega_0 t} . dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) . e^{-jk\frac{2\pi}{T_0}.t}$$

Example:



$$a_k = \frac{1}{T} \int_{-T_1}^{T_1} 1 . e^{-jk\omega_0 t} . dt = \frac{1}{T} . \frac{-1}{jk\omega_0} . e^{-jk\omega_0 t} \Big|_{-T_1}^{T_1}$$

$$= \frac{2}{k\omega_0 T} \left[\frac{e^{-k\omega_0 T_1} - e^{-jk\omega_0 T_1}}{2j} \right]$$

$$a_k = \frac{2}{k\omega_0 T} . \sin(k\omega_0 T_1)$$

$$a_k = \frac{\sin(k 2\pi T_1 / T)}{k\pi} .$$

$\Longrightarrow$ a_0'ı bulalım.

$k=0 \qquad a_0 = \frac{1}{T}\int_{-T_1}^{T_1} dt = \frac{t}{T}\Big|_{-T_1}^{T_1} = \frac{2T_1}{T}$

For $T_1 = T/4$

$a_0 = 1/2 \qquad a_{\mp 3} = 1/3\pi$

$a_{\mp 1} = 1/\pi \qquad a_{\mp 4} = 0$

$a_{\mp 2} = 0 \qquad a_{\mp 5} = 1/5\pi$

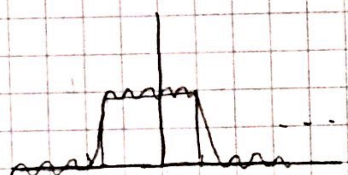
Katsayıları kullanarak fourier serisi açılımını yazalım;

$$x(t) = \sum_{-\infty}^{\infty} a_k . e^{jw_0kt}$$

$$= \frac{1}{5\pi}.e^{-jw_0 5t} + \frac{1}{3\pi}e^{-jw_0 3t} + \frac{1}{\pi}.e^{-jw_0 t}$$

$$+ \frac{1}{2} + \frac{1}{\pi}.e^{jw_0 t} + \frac{1}{3\pi}.e^{jw_0 3t} + \frac{1}{5\pi}.e^{jw_0 5t}$$

$$= \frac{1}{5\pi}\left(\frac{e^{jw_0 5t} + e^{-jw_0 5t}}{2}\right).2 + \dots$$



$$= 2\left(\frac{1}{5\pi}.\cos(5w_0t) + \frac{1}{3\pi}.\cos(3w_0t) + \frac{1}{\pi}\cos(w_0t)\dots\right) + \frac{1}{2}$$

ne kadar çok katsayı o kadar iyi kare dalga!

FOURIER TRANSFORM

$x(t) \longleftrightarrow X(jw) \qquad X(jw) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t).e^{-jwt}.dt \Rightarrow$ FOURIER TRANSFORM

$x(t) = \mathcal{F}^{-1}\{X(jw)\} = \frac{1}{2\pi}.\int_{-\infty}^{\infty} X(jw).e^{jwt}.dw \Rightarrow$ INVERSE FOURIER TRANSFORM

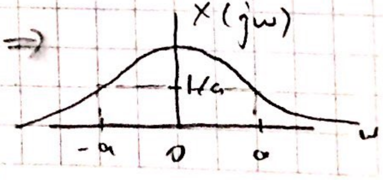
Example: $x(t) = e^{-a|t|} \quad a>0$ fourier transform?

$$X(jw) = \int_{-\infty}^{\infty} x(t).e^{-jwt}.dt = \int_{-\infty}^{0} e^{at}.e^{-jwt}.dt + \int_{0}^{+\infty} e^{-at}.e^{-jwt}.dt$$

$$= \int_{-\infty}^{0} e^{(a-jw)t}.dt + \int_{0}^{\infty} e^{(-a-jw)t}.dt$$

$$= \frac{1}{a-jw}.e^{(a-jw)t}\Big|_{-\infty}^{0} - \frac{1}{a+jw}e^{(-a-jw)t}\Big|_{0}^{\infty}$$

$$= \frac{1}{a-jw} + \frac{1}{a+jw} = \frac{2a}{a^2+w^2} \Rightarrow$$



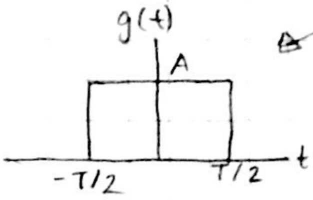
frequency

$\omega = 2\pi f \qquad d\omega = 2\pi\, df$

$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}.dt$ FOURIER TRANSFORM

$x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}.dt$ INVERSE FOURIER TRANSFORM

Example: $g(t) = A.\mathrm{rect}\left(\frac{t}{T}\right)$ fourier transform?

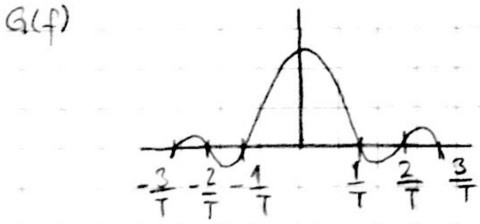


$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f.t}.dt$$

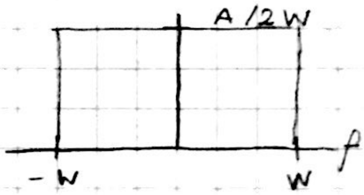
$$= \int_{-\infty}^{\infty} A.\mathrm{rect}\left(\frac{t}{T}\right).dt = \int_{-T/2}^{T/2} A.e^{-j2\pi f t}.dt$$

$$= \frac{A}{-j2\pi f}\left(e^{-j2\pi f t}\right)\Big|_{-T/2}^{T/2} = \frac{-A}{j2\pi f}\left[e^{j\pi f T} - e^{-j\pi f T}\right]$$

$$= -\frac{A}{\pi f}.\sin(\pi f T) = A.T.\mathrm{sinc}(fT)$$



INV. FOURIER TRANSFORM OF RECTANGULAR FUNC:

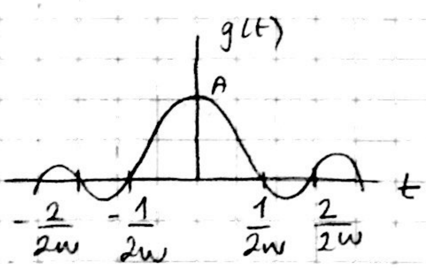


?

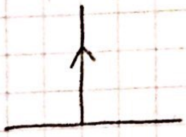
$$g(t) = \mathcal{F}^{-1}\{G(f)\} = \int_{-\infty}^{\infty} g(t).e^{-j2\pi f t}.dt$$

$$= \int_{-W}^{W} \frac{A}{2W}.e^{j2\pi f t}.df = \frac{A}{2W}.\frac{1}{j2\pi t}.e^{j2\pi f t}\Big|_{-W}^{W}$$

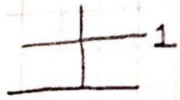
$$= A.\frac{\sin 2\pi W t}{2\pi W t} = A.\mathrm{sinc}(2Wt)$$



FOURIER TRANSFORM OF UNIT IMPULSE



$$\mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t).e^{-j2\pi f t}.dt = e^{-j2\pi f t}\Big|_{t=0} = 1$$



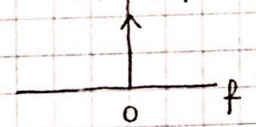
allpass

$$\mathcal{F}\{\delta(t-t_0)\} = \int_{-\infty}^{\infty} \delta(t-t_0).e^{j2\pi f t}.e^{-j2\pi f t_0} = e^{-j2\pi f t_0}.1$$

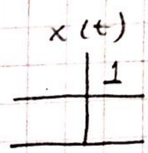
or $\mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t).e^{j2\pi f t} = e^{j2\pi f t}\Big|_{t=t_0} = e^{j2\pi f t_0}$

INVERSE FOURIER TRANSFORM OF $\delta(f)$

$X(f) = \delta(f)$



$$\mathcal{F}\{\delta(f)\} = \int_{-\infty}^{\infty} \delta(f).e^{j2\pi f}.df = e^{j2\pi f t}\Big|_{f=0} = 1$$



FOURIER TRANSFORM OF SINGLE ? SINUSOIDAL

$\mathcal{F}\{e^{j2\pi f_0 t}\} = ?$ $\quad X(f) = \delta(f-f_0) \quad x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} \delta(f-f_0)e^{j2\pi f t}.df$

$$= e^{j2\pi f t}\Big|_{f=f_0} = e^{j2\pi f_0 t}$$

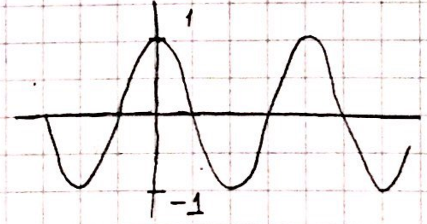
YOU MUST TO MEMORIZE IT!

$$e^{j2\pi f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} \delta(f-f_0)$$

$$e^{-j2\pi f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} \delta(f+f_0)$$

fourier transform of cosine:

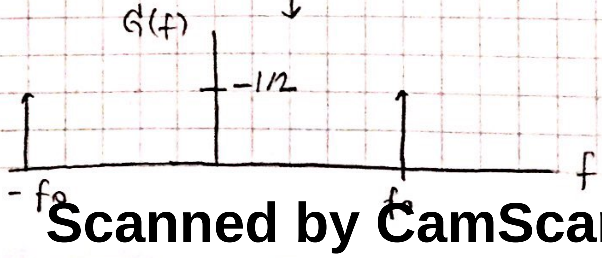
$$\mathcal{F}\{\cos(2\pi f_0 t)\} = \mathcal{F}\left\{\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right\}$$



$$= \frac{1}{2}\left[\mathcal{F}\{e^{j2\pi f_0 t}\} + \mathcal{F}\{e^{-j2\pi f_0 t}\}\right]$$

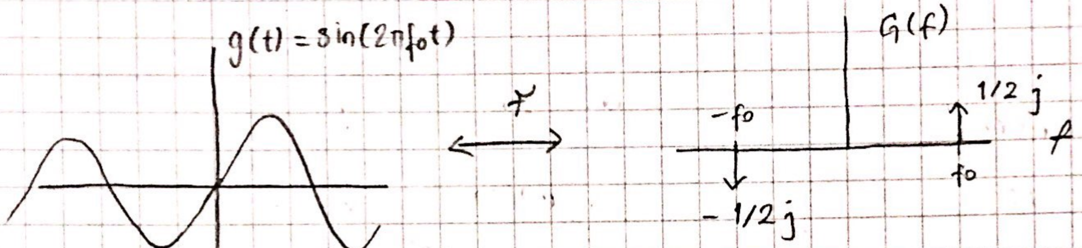
$$\Big\updownarrow \mathcal{F}$$

$$= \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$



Fourier Transform of Sine:

$$\mathcal{F}\{\sin(2\pi f_0 t)\} = \mathcal{F}\left\{\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}\right\} = \frac{1}{2j}\left[\delta(f-f_0) - \delta(f+f_0)\right]$$



Properties of fourier Transform

1. Linearity

$$x_1(t) \longleftrightarrow X_1(f)$$

$$x_2(t) \longleftrightarrow X_2(f)$$

$$a_1 \cdot x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 \cdot X_1(f) + a_2 \cdot X_2(f)$$

2. Scaling in Time

$$x(t) \longleftrightarrow X(f)$$

$$x(a.t) \longleftrightarrow \frac{1}{|a|} \cdot X\left(\frac{f}{a}\right)$$

HOW? ; $\mathcal{F}\{x(at)\} = \int_{-\infty}^{+\infty} x(at) \cdot e^{-j2\pi f t} \cdot dt$

$\tau = at$

$d\tau = a\,dt$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi f \cdot \frac{\tau}{a}} \cdot d\tau$$

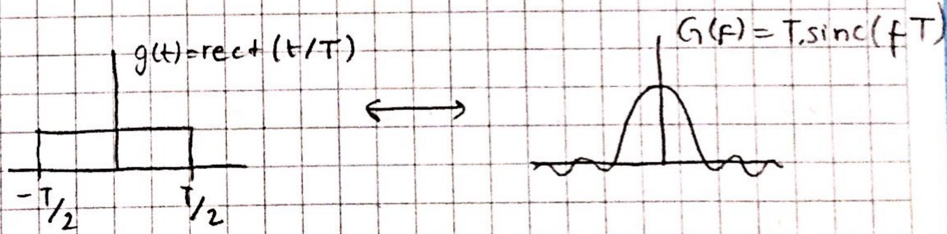
$$\frac{d\tau}{|a|} = \frac{1}{|a|} \cdot \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j2\pi \frac{f}{a}\tau} \cdot d\tau$$

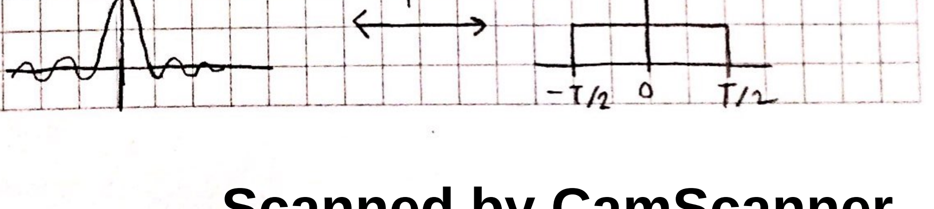
$$= \frac{1}{|a|} \cdot X\left(\frac{f}{a}\right)$$

3. Duality

$$g(t) \longleftrightarrow G(f)$$

$$G(t) \longleftrightarrow g(-f)$$





4. Time Shifting

$g(t) \longleftrightarrow G(f)$

$g(t-t_0) \longleftrightarrow e^{-j2\pi f \cdot t_0} \cdot G(f)$

How? ; proof:

$$F\{g(t-t_0)\} = \int_{-\infty}^{\infty} g(t-t_0) \cdot e^{-j2\pi f t} \cdot dt$$

$\tau = t - t_0$
$d\tau = dt$

$$= \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f(\tau + t_0)} \cdot d\tau$$

$$= e^{-j2\pi f t_0} \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f \tau} \cdot d\tau$$

$$= e^{-j2\pi f t_0} \cdot G(f)$$

5. Frequency Shifting

$g(t) \longleftrightarrow G(f)$

$e^{j2\pi f_0 t} \cdot g(t) \longleftrightarrow G(f-f_0)$

$e^{-j2\pi f_0 t} \cdot g(t) \longleftrightarrow G(f+f_0)$

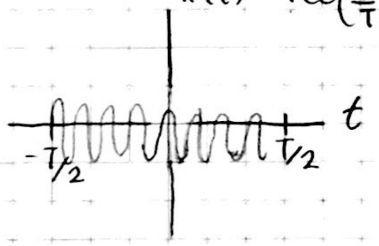
Example: $g(t) = rect\left(\frac{t}{T}\right) \cdot \cos(2\pi f_0 t) = \frac{1}{2} \cdot rect\left(\frac{t}{T}\right) \cdot \left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right]$

$$= \frac{1}{2} \cdot rect\left(\frac{t}{T}\right) \cdot e^{j2\pi f_0 t} + \frac{1}{2} \cdot rect\left(\frac{t}{T}\right) \cdot e^{-j2\pi f_0 t}$$

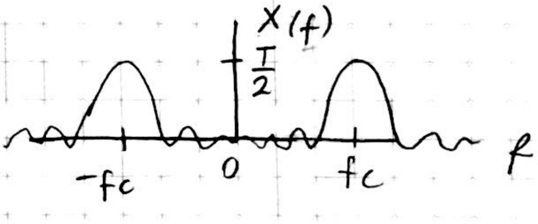
$X(f) = F\{x(t)\}$, we know that $rect\left(\frac{t}{T}\right) \xleftrightarrow{F} T \cdot sinc(fT)$

$$= \frac{1}{2} T \left[sinc(fT + f_0) + sinc(fT - f_0)\right]$$

$x(t) = rec\left(\frac{t}{T}\right) \cdot \cos(2\pi f_0 t)$



$\xleftrightarrow{F}$



6. Differention in Time Domain;

$g(t) \longleftrightarrow G(f) \qquad t \to \pm\infty \quad g(t) \to 0$

$\frac{dg(t)}{dt} \longleftrightarrow j2\pi f \cdot G(f)$

generlural: $\frac{d^n \cdot g(t)}{dt^n} \longleftrightarrow (j2\pi f)^n \cdot G(f)$

7. Modulation Theorem:

$$g_1(t) \longleftrightarrow G_1(f)$$

$$g_2(t) \longleftrightarrow G_2(f)$$

$$g_1(t).g_2(t) \longleftrightarrow G_1(f) * G_2(f) = \int_{-\infty}^{\infty} G_1(\lambda).G_2(f-\lambda).d\lambda$$

Proof: $\mathcal{F}\{g_1(t).g_2(t)\} = \int_{-\infty}^{\infty} g_1(t).g_2(t).e^{-j2\pi ft}.dt$

$$g_2(t) = \int_{-\infty}^{\infty} G_1(f').e^{j2\pi f't}.df' \quad = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(t).G_2(f).e^{-j2\pi(f-f')t}.dt.df'$$

$\lambda = f-f'$

$d\lambda = -df'$

$$= \int_{+\infty}^{-\infty} G_2(f-\lambda)\int_{-\infty}^{\infty} g_1(t).e^{-j2\pi\lambda t}.dt\,(-d\lambda)$$

$$= \int_{-\infty}^{\infty} G_2(f-\lambda)\,G_1(\lambda).d\lambda$$

$$= G_1(f) * G_2(f)$$

Example: $\mathcal{F}\{rect(\frac{t}{T}).\cos(2\pi f_c.t)\} = \mathcal{F}\{rect(\frac{t}{T})\} * \mathcal{F}\{\cos 2\pi f_c t\}$

$$= T.sinc(fT) * \frac{1}{2}.[\delta(f-f_c)+\delta(f+f_c)]$$

Bir fonksiyonu $\delta(t-t_0)$ ile çarpmak fonksiyonu "t" ekseninde t_0 kadar kaydırmak anlamına gelir.

$$= \underbrace{\frac{T}{2} sinc(fT) * \delta(f-f_c) + \frac{T}{2}.sinc(fT) * \delta(f+f_c)}_{\frac{T}{2}.sinc((f-f_c)T) + \frac{T}{2}.sinc((f+f_c)T)}$$

8. Convolution System

$$\left.\begin{array}{l} g_1(t) \xleftrightarrow{\mathcal{F}} G_1(f) \\ g_2(t) \xleftrightarrow{\mathcal{F}} G_2(f) \end{array}\right\} \; g_1(t) * g_2(t) \xleftrightarrow{\mathcal{F}} G_1(f).G_2(f)$$

Proof: $\mathcal{F}\{g_1(t) * g_2(t)\} = \mathcal{F}\{\int_{-\infty}^{\infty} g_1(\tau).g_2(t-\tau).d\tau\}$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(\tau)\,g_2(t-\tau).e^{-j2\pi ft}.dt.d\tau$$

$$= \int_{-\infty}^{\infty} g_1(\tau)\left[\int_{-\infty}^{\infty} g_2(t-\tau).e^{-j2\pi ft}.dt\right].d\tau$$

$$= \int_{-\infty}^{\infty} g_1(\tau) . G_2(f) . e^{-j2\pi f \tau} . d\tau$$

$$= G_2(f) . \int_{-\infty}^{\infty} g_1(\tau) . e^{-j2\pi f \tau} . d\tau$$

$$= G_1(f) . G_2(f)$$

Example:

$$\begin{tikzcd}
x(t) = e^{-bt}.u(t) \arrow[r] \arrow[d, "\mathcal{F}"'] & \boxed{h(t) = e^{-at}.u(t)} \arrow[r] \arrow[d, "LTI \; \mathcal{F}"] & y(t) \\
X(f) \; . & H(f) & = Y(f) \arrow[u, "\mathcal{F}^{-1}"']
\end{tikzcd}$$

$$X(f) = \frac{1}{b+j2\pi f} \qquad H(f) = \frac{1}{a+j2\pi f}$$

$$Y(f) = X(f) . H(f) = \frac{1}{b+j2\pi f} . \frac{1}{a+j2\pi f} = \frac{A}{b+j2\pi f} + \frac{B}{a+j2\pi f}$$

$$A = \frac{1}{(a+j2\pi f)(b+j2\pi f)} . (a+j2\pi f) \Big|_{f = -\frac{a}{j2\pi}} = \frac{1}{b-a}$$

$$B = \frac{1}{(b+aj2\pi f)(a+j2\pi f)} . (b+aj2\pi f) \Big|_{f = -\frac{b}{aj2\pi}}$$

$$Y(f) = \frac{1}{(b-a)} \left[\frac{1}{a+j2\pi f} - \frac{1}{b+j2\pi f} \right]$$

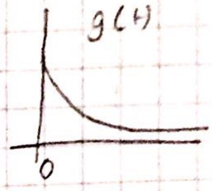
$$y(t) = \mathcal{F}^{-1}\{Y(f)\} = \frac{1}{b-a} \left[e^{-at} u(t) - e^{-bt} . u(t) \right]$$

9. Parseval's Theorem

$$g(t) \longleftrightarrow G(f)$$

$$\int_{-\infty}^{\infty} |g(t)|^2 \, dt = \int_{-\infty}^{\infty} |G(f)|^2 . df$$

Energy is preserved after transformation $|g(t)|^2 = g(t) * g^*(t)$



Example; $g(t) = e^{-at} . u(t) \quad a>0$

In time;

$$\vec{E} = \int_{-\infty}^{\infty} |g(t)|^2 . dt = \int_0^{\infty} e^{-2at} . dt = 1/2 \quad A$$

In frequency

$$G(f) = \frac{1}{a + j2\pi f} \Rightarrow \vec{E} = \int_{-\infty}^{\infty} G(f) . G^*(f) . df = \int_{-\infty}^{\infty} \frac{1}{a+j2\pi f} . \frac{1}{a-j2\pi f} . df$$

$$= \int_{-\infty}^{\infty} \frac{1}{a^2 + (2\pi f)^2} . df = \frac{1}{2\pi a} . \arctan\left(\frac{2\pi}{a} f\right)\Big|_{-\infty}^{\infty}$$

$$= 1/2a$$

FILTERING

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

LTI

$$y(t) = \int_{-\infty}^{\infty} x(\tau) . \underbrace{h(t-\tau)}_{\text{impulse response}} . d\tau$$

$$\xleftrightarrow{F} Y(f) = X(f) . \underbrace{H(f)}$$

(Transfer function
frequency response)

$H(f) = |H(f)| . \exp\{ i\theta(f) \}$

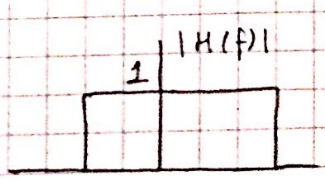
$|H(f)|$: Amplitude response

$\theta(f)$: Phase response

In logaritmic domain;

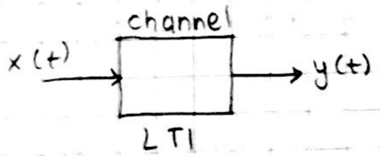
$$\ln H(f) = \underbrace{\ln |H(f)|}_{\text{Gain of the system}} + j\theta(f)$$

$20 . \log_{10} |H(f)|$ [dB] Gain in dB



$\Rightarrow$ Kazanç $= 0 = 20\log_{10} 1$

SIGNAL TRANSMISSION FROM AN LTI SYSTEM



output amplitude $|Y(f)| = |H(f)| \cdot |X(f)|$

Output phase : $\theta_y(f) = \theta_H(f) + \theta_x(f)$

[Transmission is distortionless if the input and the output
(KAYIPSIZ)
have identical wave when shapes within a multiplicative constent,]

$$y(t) = K \cdot x(t - t_d)$$

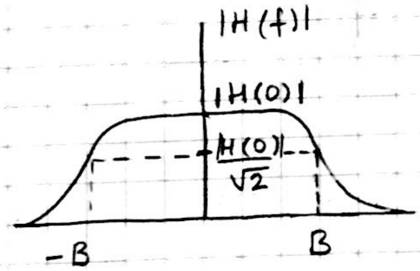
$$Y(f) = K \cdot X(f) \cdot e^{-j2\pi f t_d}$$

$$Y(f) = X(f) \cdot H(f)$$

$$H(f) = K \cdot e^{-j2\pi f \cdot t_d}$$

[This is transfer function for distortionless transmission]

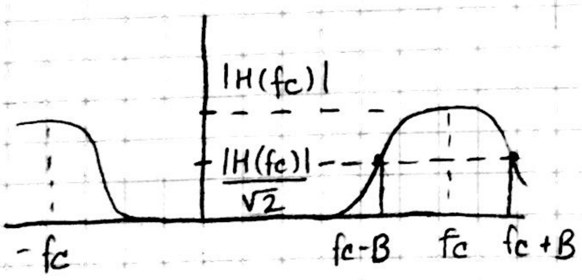
3 dB BANDWITH WITH LOW PASS



The bandwidth of a lowpass system is defined as the frequency at which the amplitude response $|H(f)|$ is $\frac{1}{\sqrt{2}}$ times its value of zero frequency.

or the frequency at which the gain drops 3 dB below its value at zero frequency.

3 dB BANDWITH: BANDPASS SYSTEM



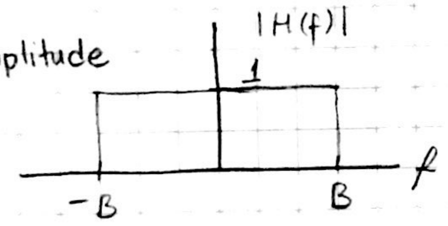
IDEAL LOW PASS FILTERS

1. The amplitude response of the filter is a constant inside the passband $-B \leq f \leq B$

2. The phase response varies *linearly* with frequency inside the passband $-B \leq f \leq B$

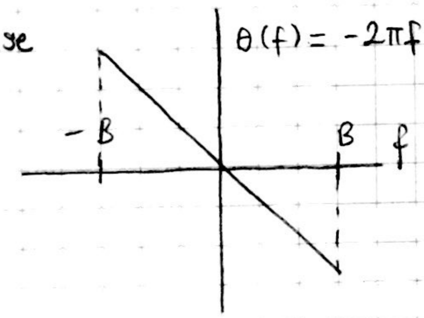
$$H(f) = |H(f)| . e^{j\theta(f)}$$

Amplitude



$$H(f) = \begin{cases} e^{-j2\pi f t_0} , & -B \leq f \leq B \\ 0 , & |f| > B \end{cases}$$

Phase



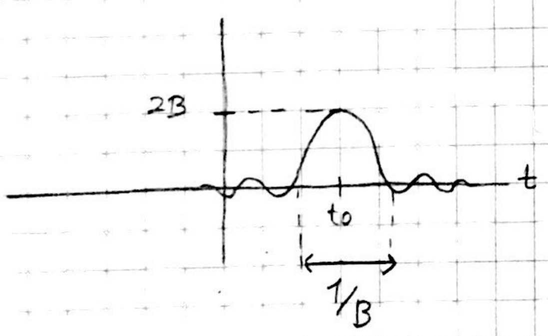
Group or envelop delay $t_g = \frac{\theta(f)}{df} = -2\pi t_0$

For ideal low-pass filter tg must be constant in $[-B, B]$

Impulse response of ideal Low-pass filter

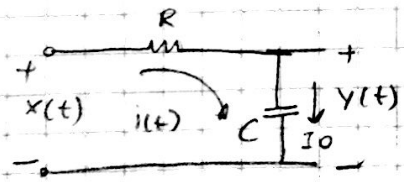
$$h(t) = \int_{-\infty}^{\infty} H(f). e^{-j2\pi f t} . dt = \int_{-B}^{B} 1. e^{-j2\pi f t_0} . e^{j2\pi f t} . df$$

$$= \frac{\sin(2\pi B(t-t_0))}{\pi(t-t_0)}^{-B} = 2B . \sin(2B(t-t_0))$$



⇒ non-casual olduğu için gerçeklenemez. Çünkü giriş işaretinin gelecekteki değerler ile çarpılması gerekir.

PRACTICAL LOW-PASS FILTER: RC CIRCUIT



$I_0 = 0$

$$RC. \frac{dy(t)}{dt} + y(t) = x(t)$$

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$H(f) = \frac{Y(f)}{X(f)}$$

$$X(f) = RC \cdot j2\pi f\, Y(f) + Y(f) = Y(f)\,(1 + RC\,j2\pi f)$$

$$\frac{X(f)}{Y(f)} = 1 + RC\,j2\pi f = H(f) = \frac{a\angle 0}{a + j2\pi f}, \quad a = 1/RC$$

$\boxed{??}$

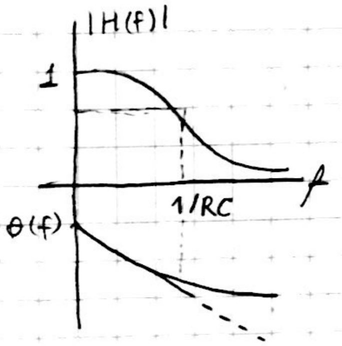
$$\frac{j2\pi f}{a} = \frac{1}{\tan(\theta)}$$

$$h(t) = \mathcal{F}^{-1}\{H(f)\} = e^{at} \cdot u(t)$$

Amplitude response $|H(f)| = \dfrac{a}{\sqrt{a^2 + (j2\pi f)^2}}$

Phase response $\theta(f) = -\arctan\left(\dfrac{2\pi f}{a}\right)$

Group or envelop delay $t_g = \dfrac{d\theta(f)}{df} = \dfrac{2\pi a}{a^2 + (2\pi f)^2}$



$\theta(f) \cong -\dfrac{2\pi f}{a}$ *Approximately linear phase.*

ENERGY SIGNAL

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} x(t) \cdot x^*(t) \cdot dt$$

$\boxed{\text{If } 0 < E_x < \infty,\ x(t) \text{ is energy signal}}$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt = \int_{-\infty}^{\infty} |X(f)|^2 \cdot df$$

$\Psi_x(f) = |X(f)|^2$ Energy Spectral Density (ESD)

$E_x = \int_{-\infty}^{\infty} \Psi_x(f) \cdot df$ ESD'nin altındaki alan energy sinyalinin değerini veriyor.

Autocorrelation function of energy signal

For real $x(t)$,

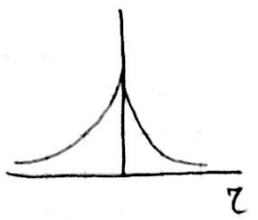
$$R_x(\tau) = \int_{-\infty}^{\infty} x(t) \cdot x(t-\tau) \cdot dt$$

Properties of Autocorrelation function :

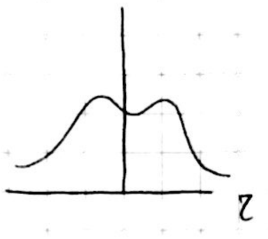
1) Symmetric in about τ $\quad R_x(\tau) = R_x(-\tau)$

2. Maximum value occurs at zero, $R_x(\tau) = R_x(0)$

3. Energy of the signal, $R_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$



autocorrelation function can applied



a.f can not applied

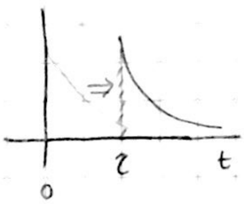
$\Rightarrow$ maximum value is not at zero

Autocorrelation function and ESD

$$\Psi_x = \int_{-\infty}^{\infty} R_x(\tau) . e^{-j2\pi f \tau} . d\tau \quad , \quad R_x = \int_{-\infty}^{\infty} \Psi_x(f) . e^{j2\pi f \tau} . df$$

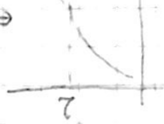
Example: $g(t) = e^{-at} . u(t)$ a) $R_g(\tau) = ?$ b) $\Psi_g(f) = ?$

a) $R_g(\tau) = \int_{-\infty}^{\infty} g(t) . \; g(t-\tau) . dt = \int_{-\infty}^{\infty} e^{-at} . \; e^{-at} . e^{+a\tau} . dt$



$\tau > 0$ $\quad \int_{\tau}^{\infty} e^{-2at} . e^{a\tau} . dt = e^{a\tau} . \left(\frac{e^{-2at}}{-2a} \right) \Big|_{\tau}^{\infty} = \frac{e^{-a\tau}}{2a}$

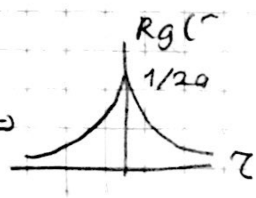
$\tau < 0 \Rightarrow$



using property of symmetry we can put $(-\tau)$ place of τ, and we can say the result is

$\frac{e^{a\tau}}{2a}$ for $\tau < 0$

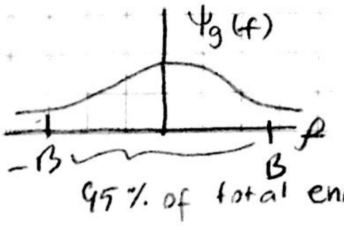
so autocorrelation function will be $R_g(\tau) = \frac{e^{-a|\tau|}}{2a} \Rightarrow$



b) $\Psi_g(\tau) = \int_{-\infty}^{\infty} R_g(\tau) . e^{-j2\pi f \tau} . d\tau = \int_{\infty}^{0} \frac{e^{a\tau}}{2a} . e^{-j2\pi f \tau} . d\tau + \int_{0}^{\infty} \frac{e^{-a\tau}}{2a} . e^{-j2\pi f \tau} . d\tau$

$$\Psi_g = \frac{1}{(2\pi f)^2 + a^2}$$

APPLICATION OF ESD ESSENTIAL BANDWIDTH



$\Psi_g(f) = \frac{1}{a^2 + (2\pi f)^2}$ $\quad E_g = 1/2a$

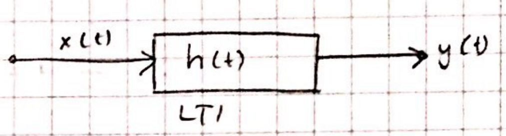
HOMEWORK

DUE: 26.10.2016

$$0.95E_g = \int_{-B}^{B} \psi_g(f).df = \int_{-B}^{B} \frac{1}{a^2+(2\pi f)^2} = \frac{1}{2\pi a}.\arctan\left(\frac{2\pi f}{a}\right)\Big|_{-B}^{B}$$

$$0{,}95\frac{1}{2a} = \frac{1}{2\pi a}.\arctan\left(\frac{2\pi B}{a}\right) \Rightarrow B = \frac{a}{2\pi}\tan(0.95\pi/2) \cong 2.02a \text{ Hz}$$

EFFECT OF FILTERING ON ESD



$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f).H(f)$$

$$|Y(f)|^2 = |X(f)|^2.|H(f)|^2$$

$$\psi_y(f) = \psi_x(f).|H(f)|^2$$

POWER SIGNAL

$$P_x = \lim_{T\to\infty}\left[\left(\int_{-T/2}^{T/2}|x(t)|^2.dt\right).\frac{1}{T}\right]$$

If $0<P_x<\infty$, $x(t)$ is a power signal.

$$x_T(t) = x(t).\mathrm{rec}\left(\frac{t}{T}\right) = \begin{cases} x(t), & -T/2 \leq t \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

$$x_T(t) \xleftrightarrow{F} X_T(f)$$

$$\int_{-\infty}^{\infty}|x(t)|^2.dt = \int_{-\infty}^{\infty}|X_T(f)|^2.df$$

$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|X(f)|^2.df = \int_{-T/2}^{T/2}\underbrace{\lim_{T\to\infty}|X(f)|^2}_{S_x(f)}.df = \int_{-T/2}^{T/2}S_x(f).df$$

Power Spectral Density (PSD)

Example:

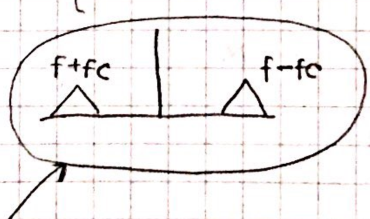
$x(t) = g(t).\cos 2\pi f_c t \quad S_x(f) = ?$

$$\cos 2\pi f_c t \xleftrightarrow{F} \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

$x(t) = g(t).\cos 2\pi f_c t \xleftrightarrow{F} X(f) = G(f) * F\{\cos 2\pi f_c t\}$

$$X(f) = G(f) * \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

$$= \frac{1}{2}G(f-f_c) + \frac{1}{2}G(f+f_c)$$



$$|X(f)|^2 = \frac{1}{4}\left[|G(f-f_c)|^2 + 2\underbrace{|G(f-f_c).G(f+f_c)|}_{0} + |G(f+f_c)|^2\right]$$

$|G(f)| = 0$ for $|f| \geq B$

$$S_x(f) = \frac{1}{4}\left[S_g(f-f_c) + S_g(f+f_c)\right] \qquad \text{If } g(t) = A,\ G(f) = A\delta(f)$$

$$S_x(f) = \frac{1}{4}\left[A^2 \cdot \delta(f-f_c) + A^2\delta(f+f_c)\right] \Rightarrow P_x = \int_{-\infty}^{\infty} \delta_x(f).df = \frac{A^2+A^2}{4} = \frac{A^2}{2}$$

cos'un gücü hesaplandı yani cosun genlik katsayısının onun karesinin yarısı

EFFECT OF FILTERING ON PSD

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$S_y(f) = S_x(f) \cdot |H(f)|^2$$

BASEBAND AND CARRIER COMMUNICATION

Baseband : In telephony , baseband is the audio band (0 – 3.5 kHz)
pair of wires, coaxial coble, optical fiber

Carrier communication: In order to enable efficient power radiation using atennas.
Anten boyutuyla frekans ters orantılıdır.

Efficient use of bandwidth

ANALOG MODULATION TECHNIQUES:

Amplitude modulation (AM)

Frequency modulation (FM)

Phase Modulation (PM)

Carrier wave : $c(t) = A_c . \cos(2\pi f_c t + \Phi_c)$

değişiklik AM — değişiklik FM — değişiklik PM

Message signal : $m(t)$

Amplitude Modulation

– Amplitude modulation

– Double side band Suppresed carrier (DSB – SC)

– Single side band (SSB)

– Vestigal side band (VSB) (Artık bir işaret mevcuttur)

göre enerjiyi daha verimli kullanır.

göre enerjiyi daha verimli kullanılır.

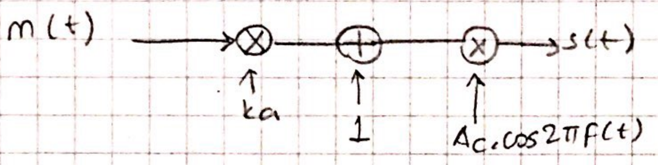
AMPLITUDE MODULATION

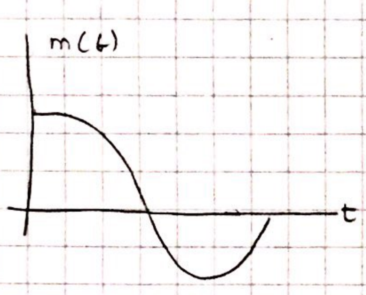
$c(t) = A_c . \cos(2\pi f_c t)$ carrier

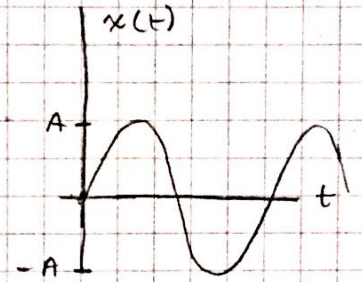
$m(t)$ message

$s(t) = A_c . [1 + k_a . m(t)] . \cos 2\pi f t$

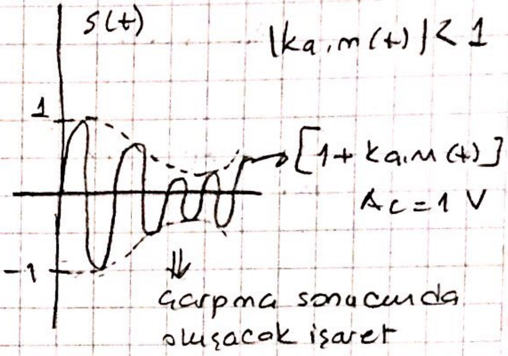
k_a : amplitude sensivity



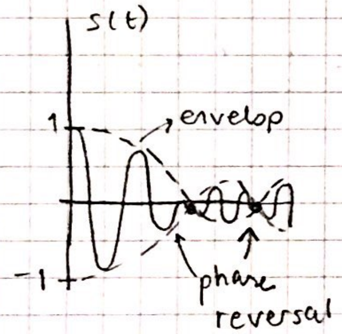




$|k_a . m(t)| < 1$



$|k_a.m(t)| > 1$ $\quad A_c = 1\ V$



Conditions in AM

1. $|K_a.m(t)| < 1$ for all t
2. $f_c >> B$ where B is the message bandwidth
3. Demodulation of AM signal can be achieved by using an envelop signal.

FREQUENCY DOMAIN DESCRIPTION OF AM

$$m(t) \longleftrightarrow M(f)$$

$$\cos 2\pi f_c t \longleftrightarrow \frac{1}{2}\{\delta(f-f_c)+\delta(f+f_c)\}$$

$$s(t) = A_c[1+k_a m(t)]\cos(2\pi f_c t) = A_c.\cos(2\pi f_c t) + A_c.k_a.m(t).\cos(2\pi f_c t)$$

$$S(f) = \mathcal{F}\{s(t)\} = A_c.\mathcal{F}\{\cos(2\pi f_c t)\} + A_c k_a \mathcal{F}\{m(t).\cos(2\pi f_c t)\}$$

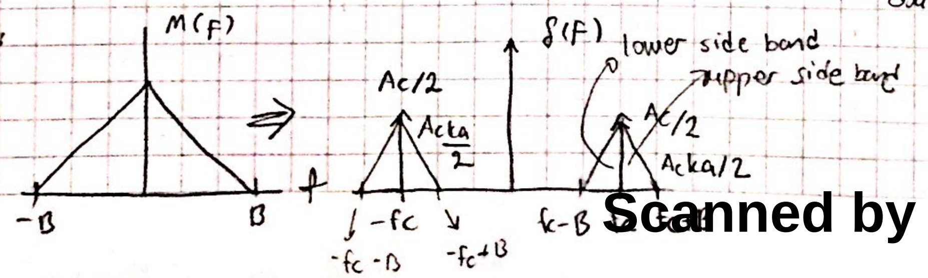
$$= A_c.\mathcal{F}\{\cos(2\pi f_c t)\} + A_c k_a M(f) * \mathcal{F}\{\cos 2\pi f_c t\}$$

$$= \frac{A_c}{2}.[\delta(f-f_c)+\delta(f+f_c)] + \frac{A_c k_a}{2} M(f) * [\delta(f-f_c)+\delta(f+f_c)]$$

$$= \frac{A_c}{2}[\delta(f-f_c)+\delta(f+f_c)] + \frac{A_c k_a}{2}[M(f-f_c)+M(f+f_c)]$$

double side'da δ fonksiyonu ihmal ediliyor. single side ise yalnızca bir sideband kullanılıyor.

Example:



Example: $m(t) = A_m . \cos(2\pi f_m t)$

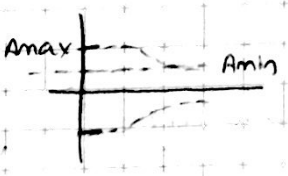
$c(t) = A_c . \cos(2\pi f_c t) \quad f_c >> f_m$

$s(t) = A_c [1 + k_a . A_m . \cos(2\pi f_m t)] . \cos(2\pi f_c t)$

$\mu = k_a . A_m \quad = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$

$\frac{A_{max}}{A_{min}} = \frac{A_c . [1+\mu]}{A_c . [1-\mu]} \Rightarrow \boxed{\mu} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

μ → modulation factor

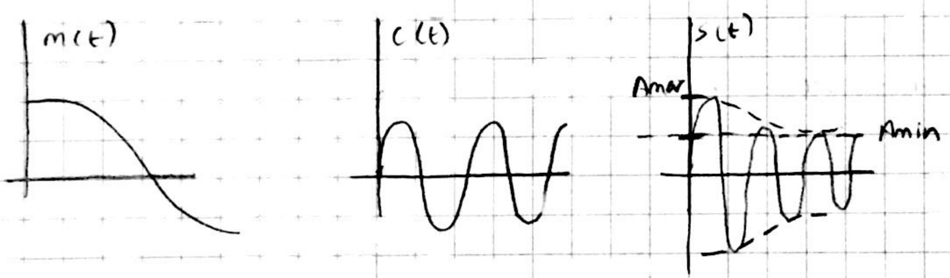


$s(t) = A_c . \cos(2\pi f_c t) + \mu . A_c . \cos(2\pi f_c t) \cos(2\pi f_m t)$

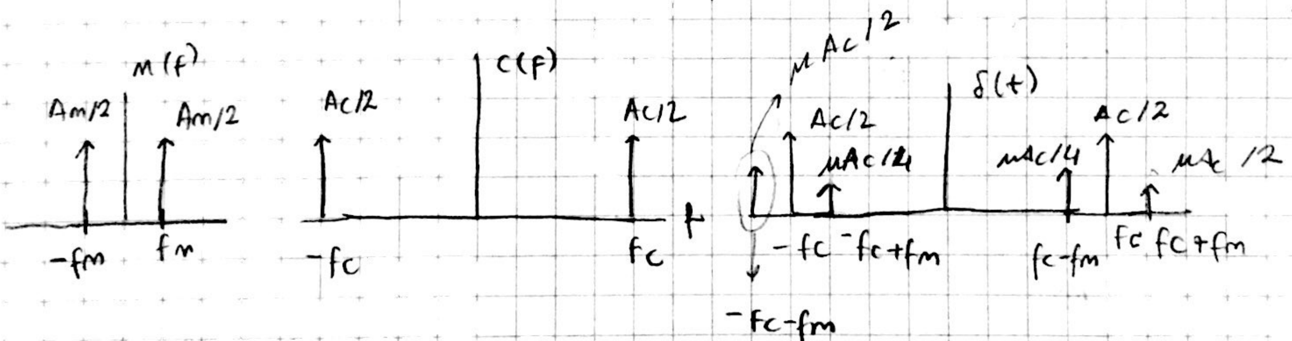
$= A_c . \cos(2\pi f_c t) + \frac{\mu . A_c}{2} . \cos(2\pi (f_c + f_m) t) + \frac{\mu . A_c}{2} . \cos(2\pi (f_c - f_m) t)$

$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{\mu A_c}{4} [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$

$+ \frac{\mu A_c}{4} [\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$



IN TIME DOMAIN



IN FREQ. DOMA

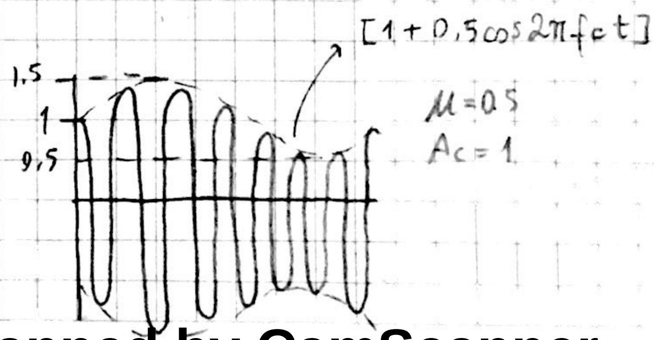
SINGLE TONE AM MODULATION

$s(t) = A_c [1 + \mu \cos 2\pi f_m . t] . \cos 2\pi f_c t$

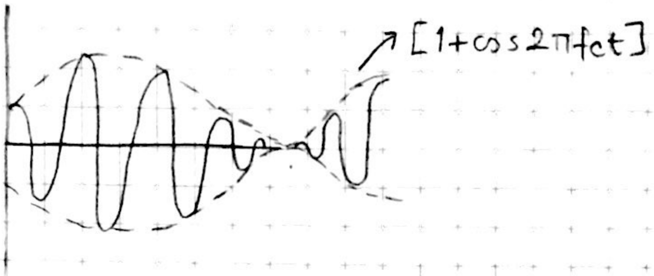
$c(t) = A_c . \cos 2\pi f_c t$

$m(t) = A_m . \cos 2\pi f_m t$

μ : modulation factor



$\mu = 1$



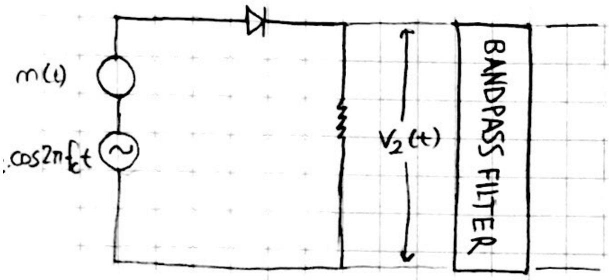
$0 < \mu < 1$ Envelop detection is avaible

$\mu = 1$ Critical modulation
Envelope detection is still avaible.

$\mu > 1$ Overmodulation
Envelope detection is no longer avaible.

GENERATION OF AM SIGNAL

square law modulation



$$v_1(t) = A_c . \cos 2\pi f_c t \quad m(t)$$

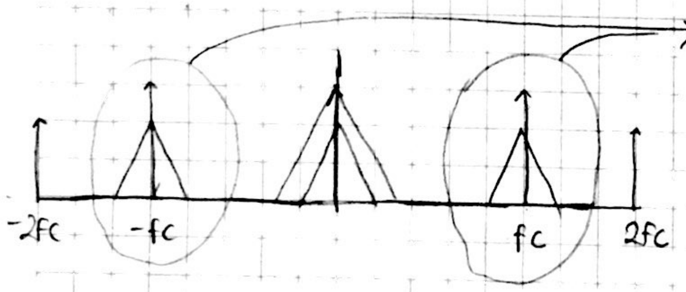
$$v_2(t) = a_1 . v_1(t) + a_2 . v_1^2(t)$$

$$= a_1 . (A_c . \cos 2\pi f_c t + m(t)) + a_2 . (A_c . \cos 2\pi f_c t + m(t))^2$$

$$= a_1 . A_c . \cos 2\pi f_c t + a_1 . m(t) + a_2 . A_c^2 . \cos^2 2\pi f_c t + 2a_2 . A_c \, m(t) . \cos 2\pi f_c t + a_2 m^2(t)$$

$$= a_1 A_c \cos 2\pi f_c t + 2a_2 A_c \, m(t) . \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \left[\frac{\cos 4\pi f_c t + 1}{2}\right] + a_2 m^2(t)$$

$$= a_1 A_c \left[1 + \frac{2a_2 . A_c(t)}{a_1 . A_c} . m(t)\right] \cos 2\pi f_c t + \ldots$$

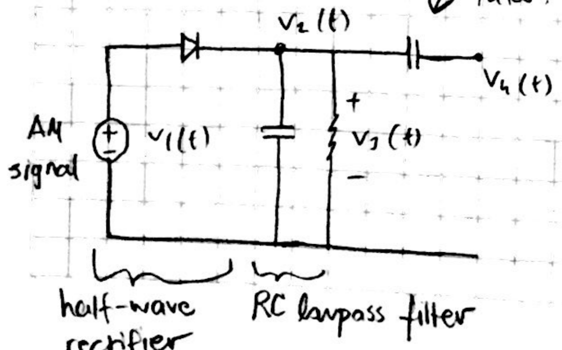


Band pass filterdan geçen işaretler ???

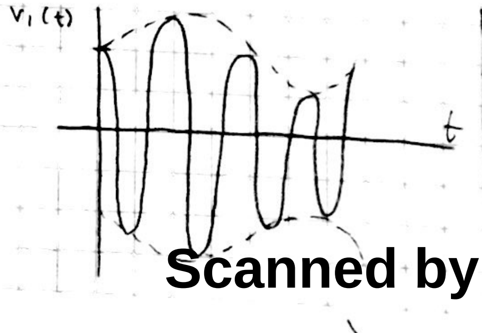
AM DEMODULATION

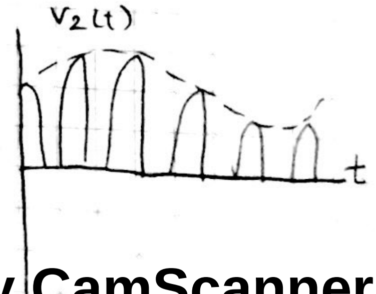
Envelope detector:

dc gerilimi tutar.

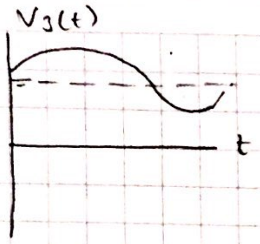


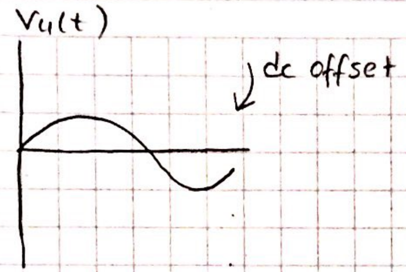
$$v_1(t) = [1 + m(t)] \cos 2\pi f_c t$$





diyottan





τ = zaman sabiti

$$\frac{1}{f_c} \ll RC \ll \frac{1}{B}$$

Bandwidth of message signal.

SIDEBAND and CARRIER POWER

$$s(t) = \underbrace{A_c \cdot \cos 2\pi f_c t}_{\text{carrier}} + \underbrace{k_a \cdot A_c\, \mu(t) \cdot \cos 2\pi f_c t}_{\text{sidebands}}$$

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{k_a A_c}{2} \cdot \left[\mu(f-f_c) + \mu(f+f_c)\right]$$

$\boxed{\text{Carrier power:}}$ $P_c = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} S_c(f) \cdot df \implies P_c = A_c^2/2$

PSD : $A_c \cdot \cos 2\pi f_c t$: $S_c(f) = \frac{A_c^2}{4}\left[\delta(f-f_c) + \delta(f+f_c)\right]^2$

$\boxed{\text{Sideband power:}}$ $P_s = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{k_a^2 \cdot A_c^2}{4}\left[\mu(f-f_c) + \mu(f+f_c)\right]^2 \cdot df$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{k_a^2 A_c^2}{4}\left[|M(f-f_c)|^2 + |M(f+f_c)|^2\right] df$$

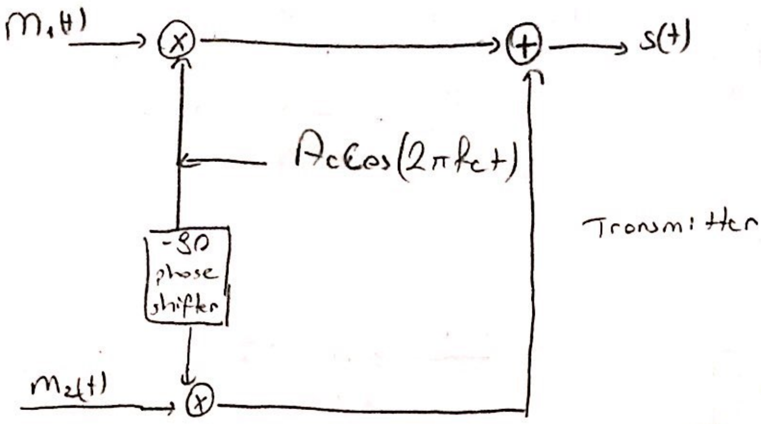
$$= \frac{k_a^2 \cdot A_c^2}{4}\left[\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |M(f-f_c)|^2 df + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |M(f+f_c)|^2 df\right]$$

$$= \frac{k_a^2 A_c^2}{4}\left[P_m + P_m\right]$$

$$P_s = \frac{k_a^2 \cdot A_c^2}{2} \cdot P_m$$

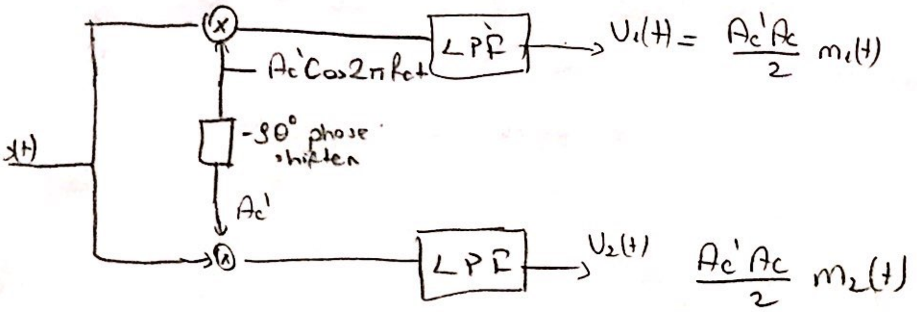
Scanned by CamScanner

Quadrature Amplitude Modulation (QAM). Analog (1) 25.10.2016



$$s(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin(2\pi f_c t)$$

$$S(f) = \frac{A_c}{2}\left[M_1(f-f_c) + M_1(f+f_c)\right] + \frac{A_c}{2j}\left[M_2(f-f_c) - M_2(f+f_c)\right]$$



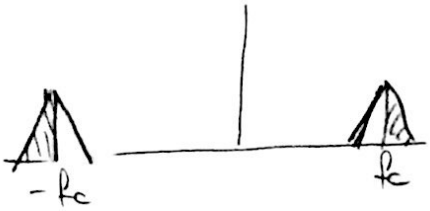
$$U(t) = s(t) \cdot A_c' \cos 2\pi f_c t$$

$$= \frac{A_c' A_c}{2}\left[m_1(t) + m_1(t) \cos(4\pi f_c t) + m_2(t) + m_2(t) \sin(4\pi f_c t)\right]$$

$$U_1(t) = \frac{A_c' A_c}{2} m_1(t)$$

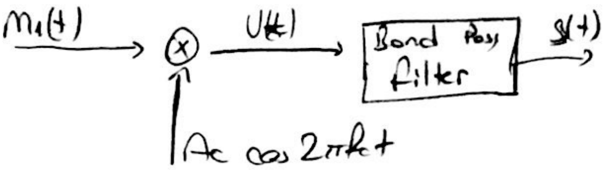
$$U_2(t) = \frac{A_c' A_c}{2} m_2(t)$$

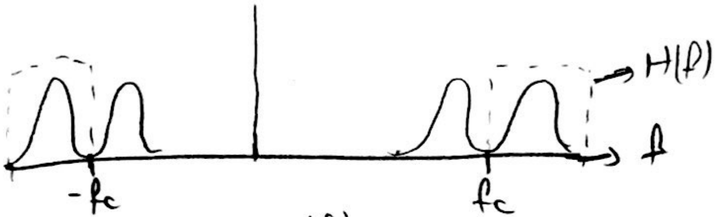
Single Side Band Modulation (SSB)



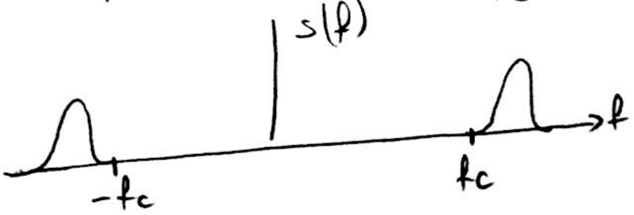
there are two common methots
1-) Selecting filtering (ses işaretlerinde kullanılır)
2-) Phase - Shifting

— Selecting filtering





Commonly used for speech signals
speech signals the freq components
below 300 Hz are not important
insan kulağı 0-900 Hz arasını duyamaz
300Hz-20kHz arasını duyabilir.
telefonda 4kHz kadar veriliyor.



— Phase Shifting (Single Tone Case)

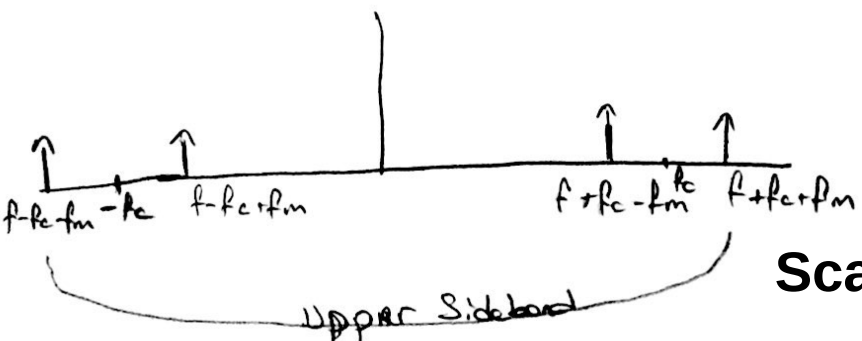
$m(t) = A_m \cos 2\pi f_m t$

$c(t) = A_m \cos 2\pi f_c t$

Double Sideband Modulation

$$S_{DSB}(t) = c(t)\, m(t) = A_c A_m \cos(2\pi f_m t) \cos(2\pi f_c t)$$

$$S_{DSB}(f) = \frac{A_c A_m}{4} \left[\delta(f - f_m + f_c) + \delta(f + f_m + f_c) + \delta(f - f_m - f_c)\, \delta(f + f_m - f_c) \right]$$



$S_{USB} = \frac{1}{2} A_c A_m \cos 2\pi (f_c + f_m) t$

Analog (2)
25.10.2016

$$S_{USB} = \frac{1}{2} A_c A_m \left[(\cos 2\pi f_c t) . \cos(2\pi f_m t) - \sin 2\pi f_c t . \sin 2\pi f_m t \right]$$

$$S_{LSB} = \frac{1}{2} A_c A_m \left[\cos(2\pi f_c t) . \cos(2\pi f_m t) + \sin 2\pi f_c t . \sin 2\pi f_m t \right]$$

Analog
26.10.2016 ①

SSB: Phase Shifting . SingleTone

$$S_{SSB}(t) = \frac{1}{2} A_c A_m \cos(2\pi f_c t) \cos(2\pi f_m t) \mp \frac{1}{2} A_c A_m \sin(2\pi f_m t) \cdot \sin(2\pi f_c t)$$

SSB: Phase shifting, periodic message signal

$$m(t) = \sum_n a_n \cos(2\pi f_n t)$$

$$c(t) = A_c \cos 2\pi f_c t$$

$$S_{SSB}(t) = \frac{1}{2} A_c \cos(2\pi f_c t) \underbrace{\sum_n \cos(2\pi f_n t)}_{m(t)} \mp \frac{1}{2} A_c \sin 2\pi f_c t \underbrace{\sum_n a_n \sin(2\pi f_n t)}_{\hat{m}(t)}$$

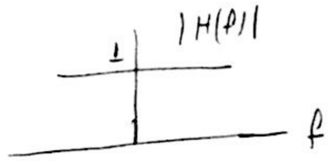
In General

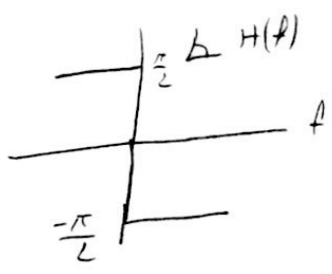
$$S_{SSB}(t) = \frac{A_c}{2} \cos(2\pi f_c t)\, m(t) \mp \frac{A_c}{2} \sin 2\pi f_c t\, \hat{m}(t)$$

$\hat{m}(t)$ is the hilbert transfor of $m(t)$

Transfer function of Hilbert transformer

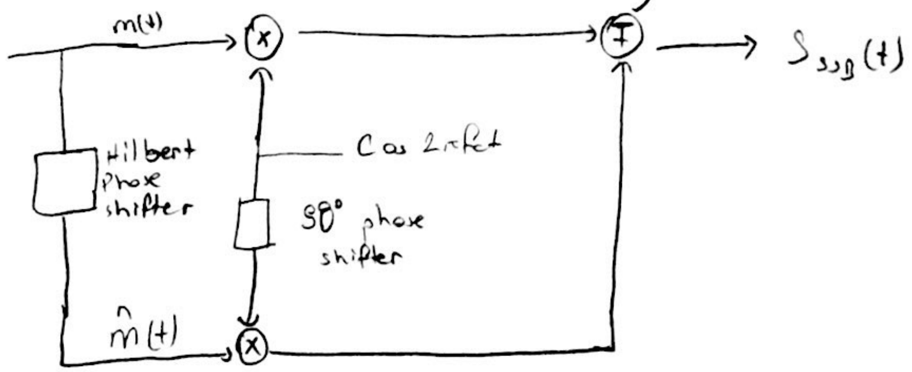
$$m(t) \longrightarrow \boxed{H(f)} \longrightarrow \hat{m}(t) \qquad H(f) = -j\,\mathrm{sign}(f) \begin{cases} -j = 1 e^{-j\frac{\pi}{2}} & f > 0 \\ j = 1 e^{j\frac{\pi}{2}} & f < 0 \end{cases}$$





$$e^{-j\pi} = \cos\frac{\pi}{2} - j \sin\frac{\pi}{2} = -j$$

for upper sideband (−)
for low sideband (+)



Hilbert Transformers : Single Tone Modulation

$m(t) = \cos(2\pi f_m t)$

$m(t) \longrightarrow \boxed{H(f)} \longrightarrow \hat{m}(t)$

LTI

$\hat{m}(t) = h(t) * m(t)$

$\hat{M}(f) = \frac{H(f)}{2}\left[\delta(f-f_m) + \delta[f+f_m)\right]$

$\hat{M}(f) = \frac{1}{2}\,\delta(f-f_m)\,e^{-j\frac{\pi}{2}} + \delta(f+f_m)\,e^{j\frac{\pi}{2}}$

$\hat{m}(t) = F^{-1}\{\hat{M}(f)\} = \frac{1}{2}\left[e^{j2\pi f_m t}\,e^{j\frac{\pi}{2}} + e^{-j2\pi f_m t}\,e^{j\frac{\pi}{2}}\right]$

$= \cos\left(2\pi f_m t - \frac{\pi}{2}\right) = \sin 2\pi f_m t$

SSB Demodulation

$S_{SSB}(t) \longrightarrow \otimes \xrightarrow{v(t)} \boxed{LPF} \longrightarrow v_o(t)$

$\uparrow$

$A_c' \cos(2\pi f_c t)$

$v(t) = S_{SSB}(t) \cdot A_c' \cos(2\pi f_c t)$

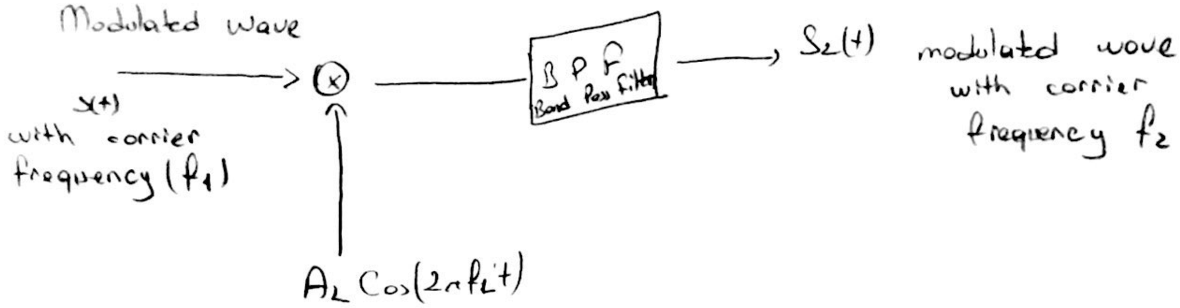
$v(t) = m(t)\, A_c A_c' \cos 2\pi f_c t \mp \hat{m}(t)\, A_c A_c' \sin(2\pi f_c t)\Big]\cos 2\pi f_c t$

$= \frac{A_c A_c'}{2}\left[m(t) + m(t)\cos(4\pi f_c t) \mp \hat{m}(t)\sin(0) \mp \hat{m}(t)\sin(4\pi f_c t)\right]$

$v_o(t) = \frac{A_c A_c'}{2}\, m(t)$

Analog
26.10.2016
2

Frequency Translation



f_2: frequency of local oscilator

$$s_1(t) = [1 + m(t)] \cos(2\pi f_1 t)$$

$$S_1(f) = \frac{1}{2}\left[\delta(f-f_1) + m(f-f_1) + \delta(f+f_1) + m(f+f_1)\right]$$

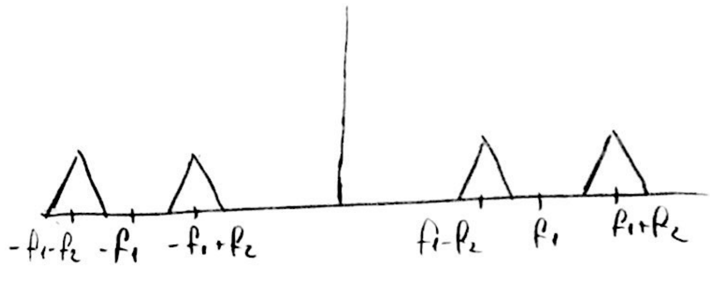
$$s_1'(t) = s_1(t) \cos 2\pi f_2 t = [1+m(t)] \cos(2\pi f_1 t) . \cos(2\pi f_2 t)$$

$$= \frac{1+m(t)}{2}\left[\cos(2\pi(f_1+f_2)t) + \cos(2\pi(f_1-f_2)t)\right]$$

$$S_1'(f) = \left[\frac{\delta(f) + m(f)}{2}\right] * \frac{1}{2}\left[\delta(f-f_1-f_2) + \delta(f+f_1-f_2) + \delta(f-f_1+f_2) + \delta(f+f_1+f_2)\right]$$

$$= \frac{1}{4}\left[\delta(f-f_1-f_2)\ m(f-f_1-f_2)\right] + \frac{1}{4}\left[\delta(f+f_1+f_2)\ m(f+f_1+f_2)\right] + \frac{1}{4}\left[\delta(f-f_1+f_2)\ m(f-f_1+f_2)\right]$$

$$+ \frac{1}{4}\left[\delta(f-f_1+f_2) . m(f-f_1+f_2)\right]$$



for upconversion $f_2 = f_1 + f_c$

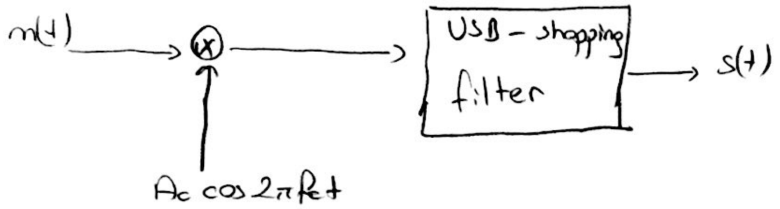
for down-conversion $f_2 = f_1 - f_c$

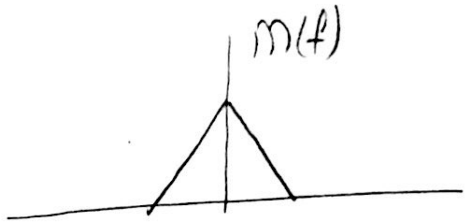
band geçire filtre ile istediğin

mesaj işaretini seçebilirsin

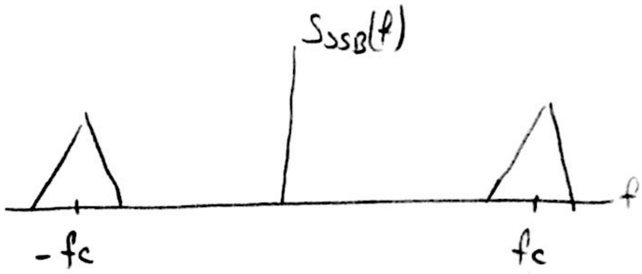
Vestigial Sideband Modulation (VSB)

- The spectra of wideband signals contain significant low frequency Ex: Television, video signals
- That make it impractical to use SSB modulation
- Instead of complately removing a sideband a trace or vestige of that band is transmitted

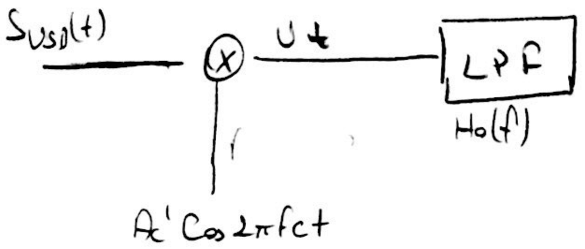


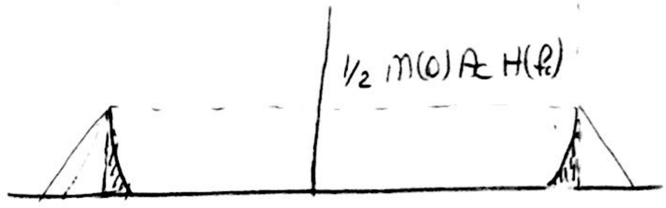


$$s_{VSB}(t) = \left[m(t)\, A_c \cos 2\pi f_c t\right] * h(t)$$



$$S_{VSB}(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \cdot H(f)$$





$$U(t)\ S_{VSB}(t)\ A_c' \cos 2\pi f_c t$$

$$U(f) = \frac{A_c'}{2}\left[S_{VSB}(f-f_c) + S_{VSB}(f+f_c)\right]$$

$$U(f) = \frac{A_c A_c'}{4}\left[M(f-2f_c) + M(f)\right] H(f-f_c) + \frac{A_c A_c'}{4}\left[M(f) + M(f+2f_c)\right] H(f+f_c)$$

Analog
26.10.2016
3

$$U_o(f) = \frac{A_c'}{4}\left[M(f-2f_c) + M(f)\right] H(f-f_c)\, H_o(f) + \frac{A_c'}{4}\left[M(f+2f_c) + M f\right] H(f+f_c) \cdot H_o(f)$$

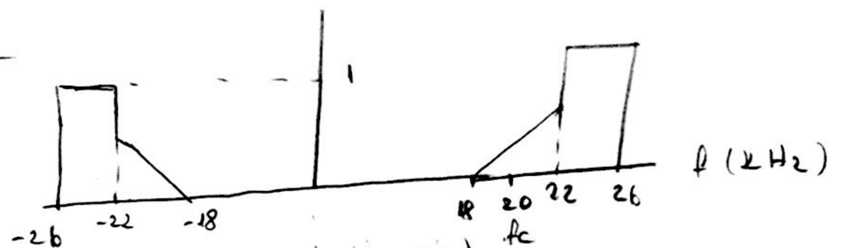
$$U_o(f) = \frac{A_c'}{4}\left[\underbrace{H(f-f_c)\, H_o(f) + H(f+f_c)\, H_o(f)}_{=1}\right] M(f)$$

$\left[H(f-f_c) \cdot H_o(f) + H(f+f_c) \cdot H_o(f)\right] = 1$ olursa Vestige side band filter yapmış oluruz.

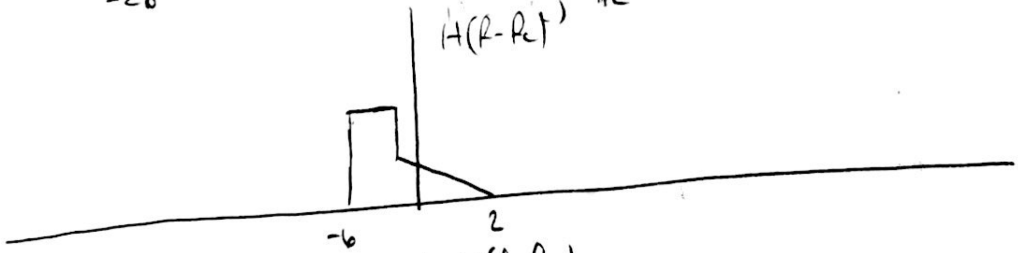
$$H_o(f) = \frac{1}{H(f-f_c) + H(f+f_c)} \qquad -B < f < B$$

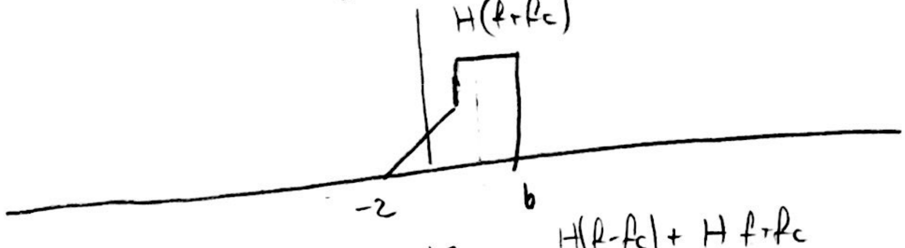
B: band wide of message signal

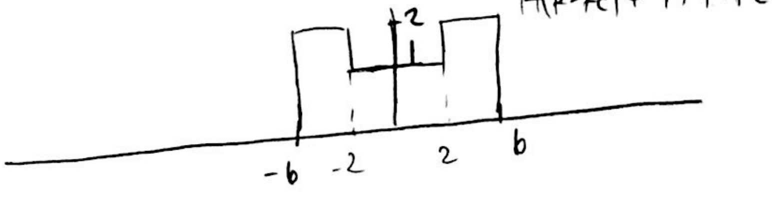
ex:

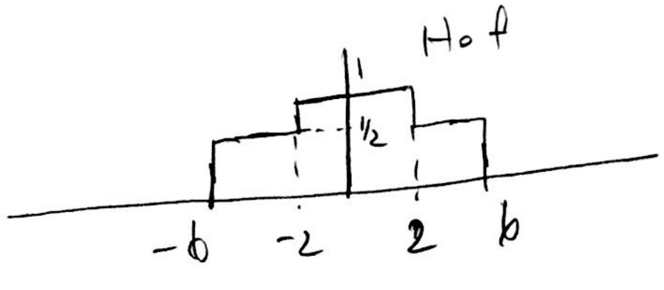


$f_c = 20$







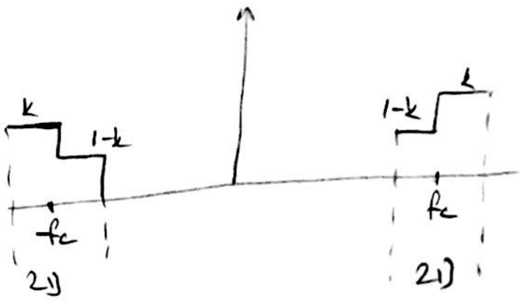


Analog
02.11.2016
1

ex

$m(t) = A_m \cos(2\pi f_m t)$
$c(t)\quad A_c \cos(2\pi f_c t)$

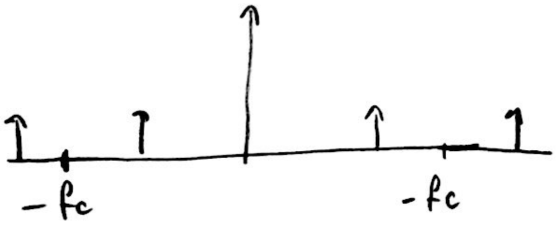
$$H(f) = \begin{cases} k & |f| \geq f_c \\ 1-k & |f| < f_c \end{cases}$$

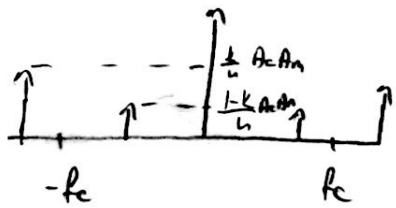


$$s(t) = [m(t) \cdot c(t)] * h(t)$$

$$S(f) = [M(f) * C(f)] \cdot H(f)$$

$$= \frac{1}{4} A_c A_m \left[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)) + \delta(f - f_c - f_m) + \delta(f + f_c - f_m) \right] H(f)$$





$$S(f) = A_c A_m \frac{1}{4} (1-k) \left[\delta(f - f_c + f_m) + \delta(f + f_c + f_m) \right] +$$

$$A_c A_m \frac{1}{4} \left[\delta(f - f_c - f_m) + \delta(f + f_c - f_m) \right]$$

$$s(t) = \frac{1}{2} k A_c A_m \cos 2\pi(f_c + f_m) + \frac{1}{2}\, 1-k\, A_c A_m \cos(2\pi(f_c - f_m))$$

$$s(t) = \frac{1}{2} A_c A_m \cos(2\pi f_c t) \cdot \cos 2\pi f_m t + \frac{1}{2} (1-2k) A_c A_m [\sin(2\pi f_c t) \cdot \sin 2\pi f_m t]$$

① $k = \frac{1}{2}$ => s(t) reduce to DSB - SC

$k = 0$ => s(t) reduce to lower Single side band modulation

② $k = 1$ => s(t) reduc to upper Single side ban modulation

③ $0 < k < \frac{1}{2}$ s(t) for which the attenuated version of upper sid frequency defines the vestige of s(t)

④ $\frac{1}{2} < k < 1$ for which the attenuated version of the lower side frequency defines the vestige of s(t)

Summary of Amplitude Modulation

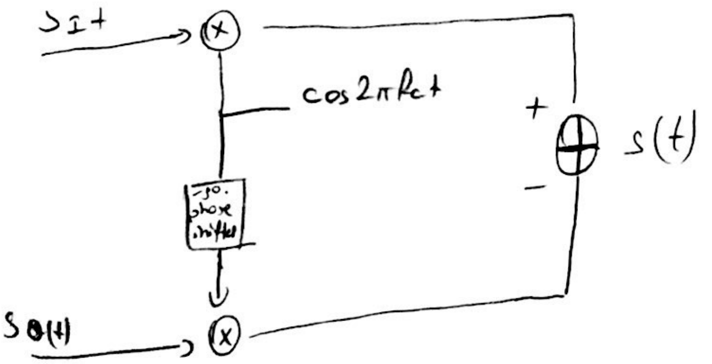
Generic linear Modulated Wave

$s(t) = S_I(t) \underbrace{\cos 2\pi f_c t}_{\text{Carrier}} - S_Q(t) \sin 2\pi f_c t$

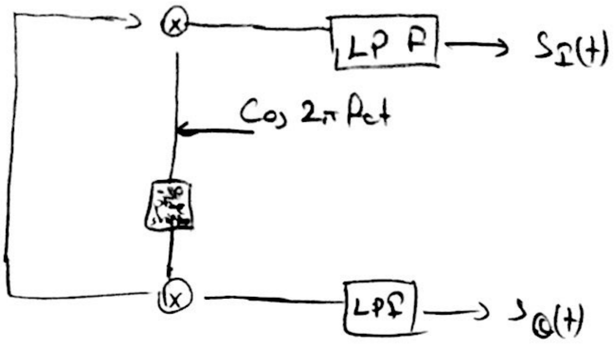
Quatrature -phase version of carrier

$S_I(t)$ = I- phase complament

$S_Q(t)$ = Quadiature - phase



Transmitter



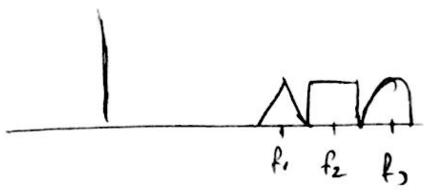
Receiver

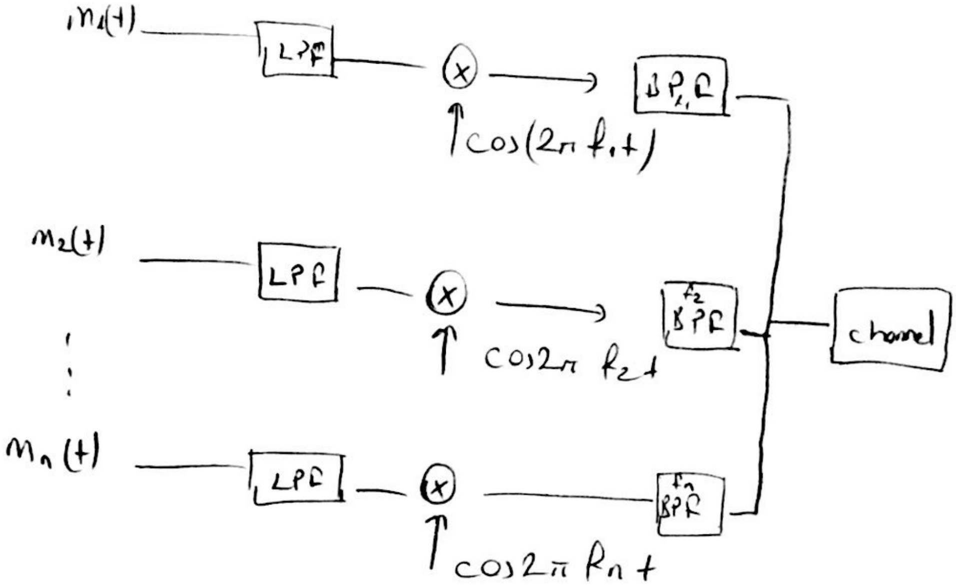
Type of mod	$s_I(t)$	$s_Q(t)$	
AM	$1 + k_a m(t)$	0	
DSB-SC	$m(t)$	0	
SSB (upper side band)	$\frac{1}{2} m(t)$	$\frac{1}{2} \hat{m}(t)$	$\hat{m}(t)$: Hilbert transform of $m(t)$
SSB lower side band	$\frac{1}{2} m(t)$	$-\frac{1}{2} \hat{m}(t)$	
VSB of lower side band	$\frac{1}{2} m(t)$	$\frac{1}{2} m'(t)$	$m'(t)$: response of filter with transfer function
VSB of upper side band	$\frac{1}{2} m(t)$	$-\frac{1}{2} m'(t)$	$H_Q(f) = -j\left[H(f+f_c) - H(f-f_c)\right]$

Frequency Division Multiplexing (FDM)

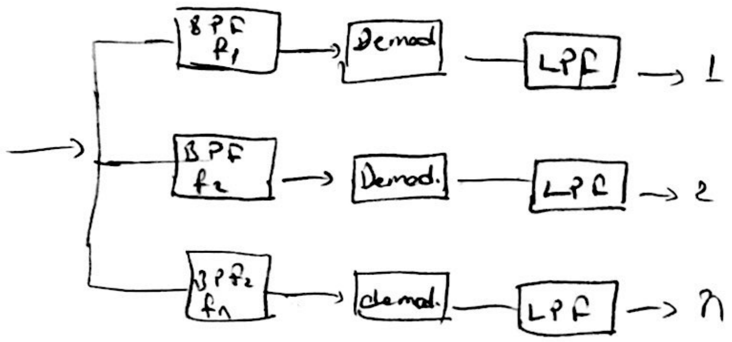
- It's used to transmit a number of signals over the same channel

- Seperating the signals in frequency is refered to as frequency division multiplexing





transmitter



Reciever

There are three operation before demodulation

1-) Carrier frequency tunning to select the desired signal

2-) filtering to seperate that signal from the others recieved along with it

3-) amplification to compansate for transmission loss

High gain tunable band pas emplifier in RF frequency is expensive and difficult

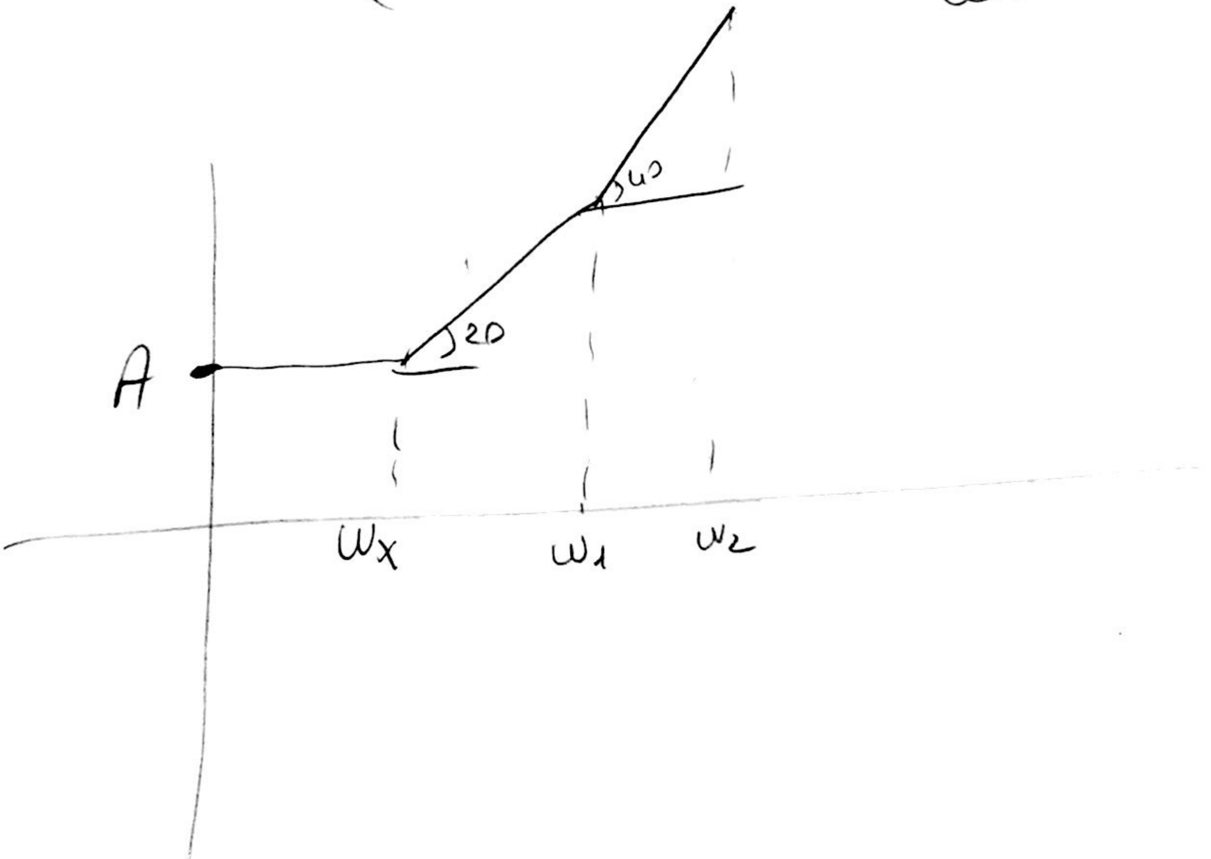
Analog

Edwin Amstrong devised the superheterodyne 02.11.2016 receiver to overcame those problems in 1918 during Wold War I

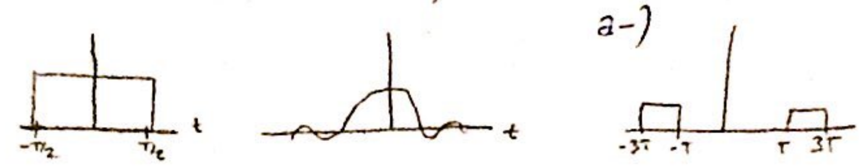
$$A \frac{(j\omega_x l}{\left(1+\frac{1}{\omega_2 C}\right)+1+\frac{1}{\omega_3 C}}$$

~~×C8~~ $x =$ zero

$$\omega_x < \omega_2 < \omega_3$$



HW 1-1 $x(t) = \mathrm{rect}\left(\frac{t}{T}\right) \longleftrightarrow X(f) = T\,\mathrm{sinc}(fT)$



a-)

$$x_a(t) = \mathrm{rect}\left(\frac{t-2T}{2T}\right) + \mathrm{rect}\left(\frac{t+2T}{2T}\right)$$

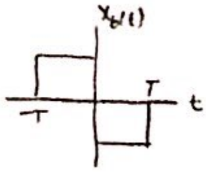
$$X_a(f) = \mathcal{F}\left\{\mathrm{rect}\left(\frac{t-2T}{2T}\right)\right\} + \mathcal{F}\left\{\mathrm{rect}\left(\frac{t+2T}{2T}\right)\right\}$$

$$= \mathcal{F}\left\{\mathrm{rect}\left(\frac{t}{2T}\right)\right\} e^{-j2\pi f 2T} + \mathcal{F}\left\{\mathrm{rect}\left(\frac{t}{2T}\right)\right\} e^{j2\pi f 2T}$$

$$= 2T\,\mathrm{sinc}(2fT)\left\{e^{j4\pi f T} + e^{-j4\pi f T}\right\}$$

$$X_a(f) = 4T\,\mathrm{sinc}(2fT)\cos(4\pi f T)$$

b-)



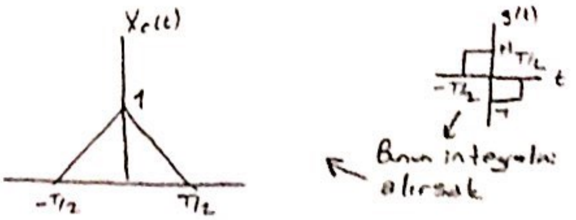
$$x_b(t) = \mathrm{rect}\left(\frac{t+T/2}{T}\right) - \mathrm{rect}\left(\frac{t-T/2}{T}\right)$$

$$X_b(f) = \mathcal{F}\left\{\mathrm{rect}\left(\frac{t+T/2}{T}\right) - \mathcal{F}\left\{\mathrm{rect}\left(\frac{t-T/2}{T}\right)\right\}\right\}$$

$$= \mathcal{F}\left\{\mathrm{rect}\left(\frac{t}{T}\right)\right\}\left(e^{j2\pi f T/2} - e^{-j2\pi f T/2}\right)$$

$$= T\,\mathrm{sinc}(fT)\,2j\sin(\pi f T)$$

c-)



$$x_c(t) = \int_{-\infty}^{t} g(\tau)\,d\tau$$ ← **I. Yöntem**

$$g(t) = \mathrm{rect}\left(\frac{2(t+T/4)}{T}\right) - \mathrm{rect}\left(\frac{2(t-T/4)}{T}\right)$$

If $G(0) = 0$ $\quad X_c(f) = \frac{1}{j2\pi f} G(f)$

$$G(0) = \int_{-\infty}^{\infty} g(t)\,dt = 0$$

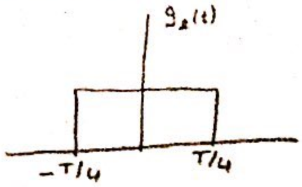
$$G(f) = \mathcal{F}\left\{\mathrm{rect}\left(\frac{2t}{T}\right)\left(e^{j2\pi f T/4} - e^{-j2\pi f T/4}\right)\right\}$$

$$= jT\,\mathrm{sinc}\left(\frac{fT}{2}\right)\sin\left(\frac{\pi f T}{2}\right)$$

$$X_c(f) = \frac{jT}{j2\pi f}\,\mathrm{sinc}\left(\frac{fT}{2}\right)\sin\left(\frac{\pi f T}{2}\right)$$

$$X_c(f) = \frac{T^2}{4}\,\mathrm{sinc}^2\left(\frac{fT}{2}\right)$$

II. Ysnten



$g_2(t) = \text{rect}\left(\frac{2t}{T}\right)$

$X_c(t) = g_2(t) * g_2(t)$

$X_c(f) = |G_2(f)|^2$

$G_2(f) = \mathcal{F}\{\text{rect}(\frac{2t}{T})\} = \frac{1}{2} X(\frac{f}{2}) = \frac{T}{2} \text{sinc}\left(\frac{fT}{2}\right)$

$X_c(f) = \frac{T^2}{4} \text{sinc}^2\left(\frac{fT}{2}\right)$

2-) $x(t) \to \boxed{h(t)} \to y(t) = a e^{-bt} u(t)$ $\qquad X(f) = \frac{a+j2\pi f}{b+2\pi j f}$

a-) $H(f) = ?$ $\qquad H(f) = \frac{Y(f)}{X(f)}$

$Y(f) = \int_{-\infty}^{\infty} a e^{-bt} u(t) e^{-j2\pi f t} dt = a \int_0^{\infty} e^{-(b+j2\pi f)t} dt$

$= \frac{a}{-(b+2\pi f j)} \cdot e^{-(b+j2\pi f)t} \Big|_0^{\infty} = \frac{a}{b+j2\pi f}$

$H(f) = \frac{a}{b+j2\pi f} \frac{b+j2\pi f}{a+j2\pi f} = \frac{a}{a+j2\pi f}$

b-) $|H(f)| = \frac{|a|}{|a+j2\pi f|} = \frac{a}{\sqrt{(a+j2\pi f)(a-2\pi j f)}} = \frac{a}{\sqrt{a^2+(2\pi f)^2}}$

c-) $\theta_H(f) = \angle a - \angle (a+j2\pi f)$

$= 0 - \arctan\left(\frac{2\pi f}{a}\right) = -\arctan\left(\frac{2\pi f}{a}\right)$

d-) $t_g = \frac{d\theta_H(f)}{df} = \frac{2\pi a}{a^2+(2\pi f)^2}$

e-) $|H(0)| = 1$ Low pass

f-) $\frac{|H(0)|}{\sqrt{2}} = |H(f_3)| = \frac{1}{\sqrt{2}} = \frac{a}{\sqrt{a^2+(2\pi f_3)^2}}$ $\qquad 2a^2 = a^2 + (2\pi f_3)^2$ $\qquad f_3 = \frac{a}{2\pi}$

g-) Essential bandwidth E_H %99

$E_H = \int_{-\infty}^{\infty} |H(f)|^2 df$

$E_H = \int_{-\infty}^{\infty} \frac{a^2}{a^2+(2\pi f)^2} df = a^2 \frac{1}{2\pi a} \arctan\left(\frac{2\pi f}{a}\right)\Big|_{-\infty}^{\infty} = \frac{a}{2}$

Bulduğumuz enerjinin %99'u

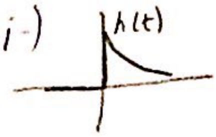
$0{,}99 E_H = \int_{-B}^{B} |H(f)|^2 df$

$0{,}99\frac{a}{2} = \frac{a}{2\pi}\left[\arctan\left(\frac{2\pi B}{a}\right) - \arctan\left(-\frac{2\pi B}{a}\right)\right]$

$B = \frac{a}{2\pi} \tan\left(\frac{0{,}99\pi}{2}\right)$

h-) $h(t) = \mathcal{F}^{-1}\{H(f)\} = \mathcal{F}^{-1}\left\{\frac{a}{a+j2\pi f}\right\} = a e^{-at} u(t)$

i-)



since $h(t) = 0$ for $t < 0$, causal