Lecture #12 Overview

- Angle Modulation: Phase Modulation and Frequency Modulation

PM and FM Signal Representation

- Phase Modulation and Frequency Modulation are special cases of Angle Modulated signalling. In angle modulated signalling the complex envelope is

$$g(t) = A_c e^{j\theta(t)}$$

- Note that this is in polar form so we can immediately say what are the amplitude modulation $R(t)$ and the phase modulation $\theta(t)$

- The amplitude modulation is

$$R(t) = |g(t)| = A_c = \text{constant}$$

- The phase modulation is simply $\theta(t)$ and for angle modulated signals is a linear function of the modulating signal $m(t)$.

PM & FM Signals Representation (cont.)

- The resulting angle modulated signal is

$$s(t) = \Re\{A_c e^{j\theta(t)} e^{j\omega_c t}\} = A_c \cos[\omega_c t + \theta(t)]$$

- For PM the phase is directly proportional to the modulation (or message signal amplitude)

$$\theta(t) = k_p m(t)$$

where the constant k_p is the phase sensitivity having units of radians per volt (if $m(t)$ is a voltage waveform).

- For FM the phase is proportional to the integral of $m(t)$.

$$\theta(t) = k_f \int_{-\infty}^{t} m(\sigma) d\sigma$$

where the frequency deviation constant k_f has units of radians per volt-second.

FM on a PM Signal and Vice-Versa

- If a PM signal is modulated by $m_p(t)$ then there is also FM on the signal corresponding to a different modulation waveshape

$$m_f(t) = \frac{k_p}{k_f}\left[\frac{dm_p(t)}{dt}\right]$$

- If a FM signal is modulated by $m_f(t)$ then the corresponding phase modulation on this signal is

$$m_p(t) = \frac{k_f}{k_p}\int_{-\infty}^{t} m_f(\sigma)d\sigma$$

- So, a FM signal can be generated using a circuit with an integrator and a phase modulator and a PM signal can be generated using a circuit with a differentiator and a frequency modulator.

Instantaneous Frequency

- If a bandpass signal is represented by

$$s(t) = R(t)\cos\psi(t)$$

where $\psi(t) = \omega_c t + \theta(t)$, then the instantaneous frequency (hertz) of $s(t)$ is

$$f_i(t) = \frac{1}{2\pi}\omega_i(t) = \frac{1}{2\pi}\left[\frac{d\psi(t)}{dt}\right] = \frac{1}{2\pi}\left[\omega_c + \frac{d\theta(t)}{dt}\right] = f_c + \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$$

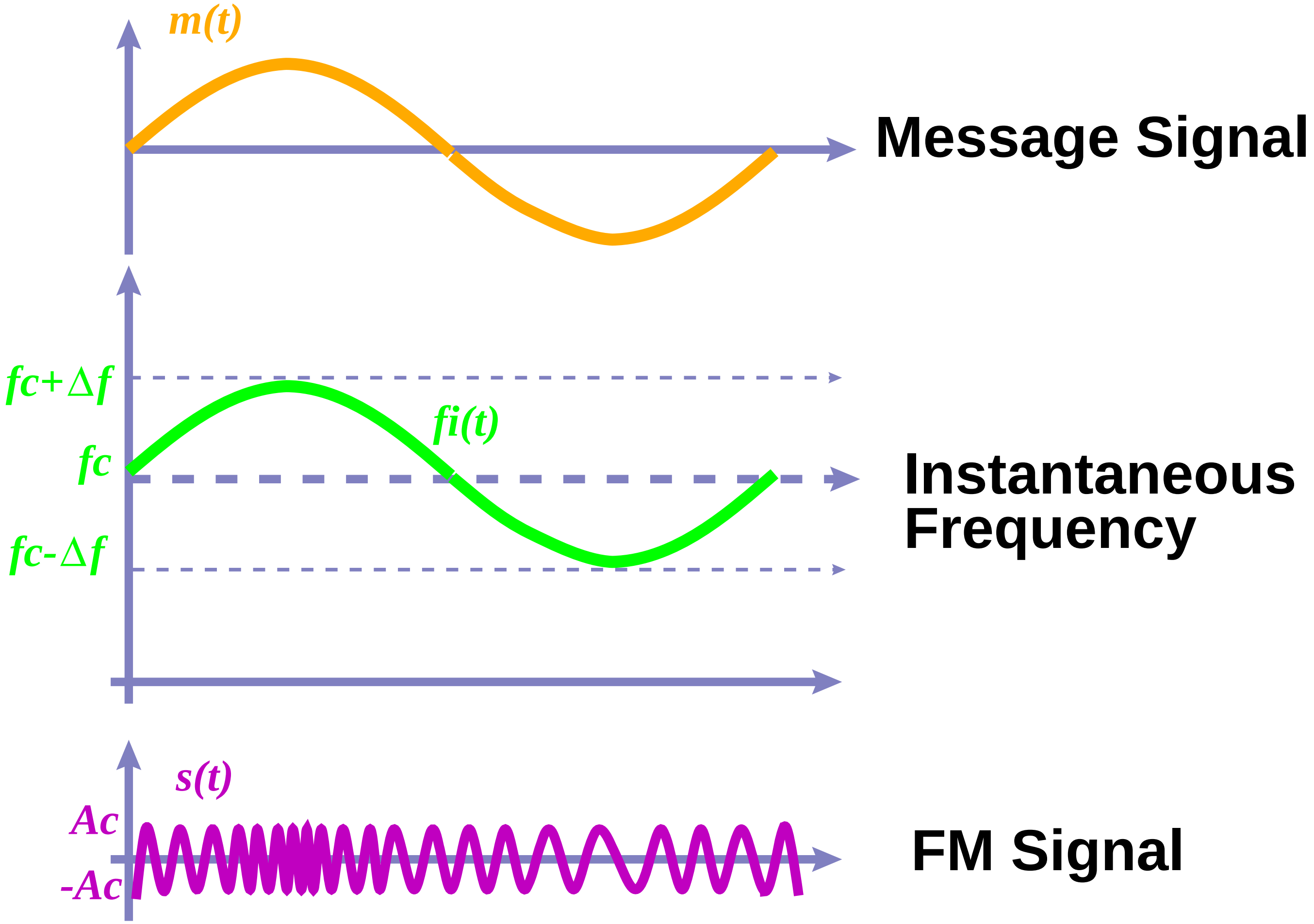
Instantaneous Frequency cont.

- Thus, for the case of FM, the instantaneous frequency is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d(2\pi f_c t + k_f \int_{-\infty}^{t} m(\sigma)d\sigma)}{dt} = f_c + \frac{1}{2\pi}k_f m(t)$$

- That is, the instantaneous frequency of a FM signal varies about the carrier f_c in a manner proportional to the modulating signal $m(t)$
- This is the fundamental principle behind FM.

Fundamental Principle of FM



More Frequency Stuff...

- The frequency deviation from the carrier frequency, at any given time, is

$$f_d(t) = f_i(t) - f_c = \frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]$$

- The peak frequency deviation is

$$\Delta F = \max\left\{\frac{1}{2\pi}\left[\frac{d\theta(t)}{dt}\right]\right\}$$

where ΔF is positive.

- For FM signalling the peak frequency deviation is related to the peak modulating voltage by

$$\Delta F = \frac{1}{2\pi}k_f V_p \quad \text{where} \quad V_p = \max[m(t)]$$

Further Observations for FM Signals

- An increase in the amplitude, V_p, of $m(t)$ will increase ΔF
- An increase in the amplitude, $V_{p,}$ of $m(t)$ will increase the bandwidth of the FM signal.
- An increase in the amplitude, $V_{p,}$ of $m(t)$ will not increase the average power level of the FM signal which will remain at $A_c^2/2$.
- That is, spectral components will appear further and further away from f_c and spectral components near f_c will decrease in magnitude (since the total power remains constant).
- Compare with AM signalling...

More Phase Stuff…

- The peak phase deviation may be defined by

$$\Delta\theta = \max[\theta(t)]$$

which, for PM, is related to the peak modulating voltage V_p *by*

$$\Delta\theta = k_p V_p$$

where $V_p = \max[\theta(t)]$