GEBZE YÜKSEK TEKNOLOJİ ENSTİTÜSÜ

ELEKTRONİK MÜHENDİSLİĞİ BÖLÜMÜ

ELM 361
ANALOG COMMUNICATION SYSTEMS

ÖDEV SORULARI ve ÇÖZÜMLERİ

Hazırlayan

Hakan ÖZDEMİR

İstek ve önerileriniz için bize ulaşın
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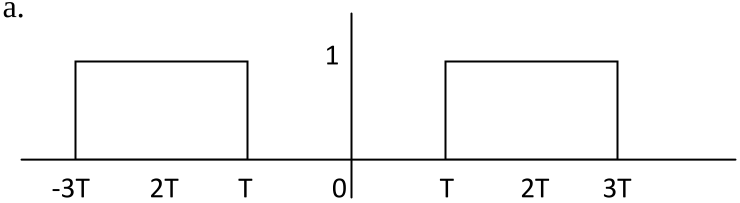
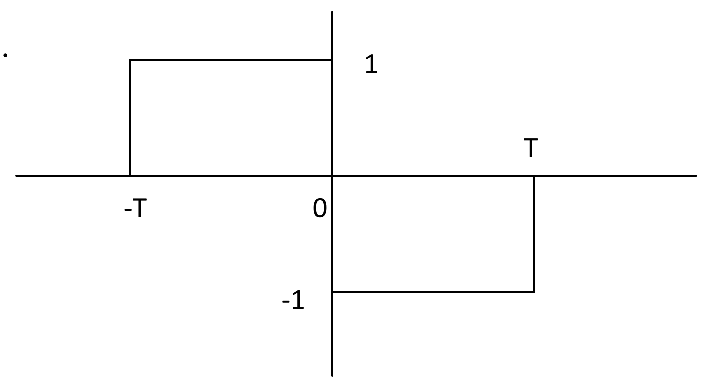
ELM361 Analog Communication Systems
Homework #1
Due: 05.11.2013

1. A signal $x(t) = \text{rect}\left(\frac{t}{T}\right)$ has the following Fourier transform $X(f) = T sinc(\pi f T)$. Determine the Fourier transform of the following signals using $X(f)$:

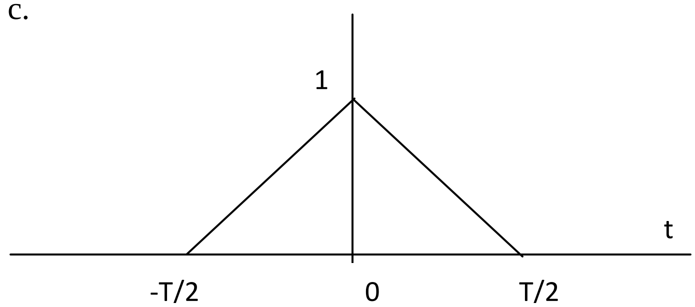
   a.



   b.



   c.



2. Consider a filter with impulse response $h(t)$ and transfer function $H(f)$. If we apply the input signal $x(t)$ whose Fourier spectrum is $X(f) = \dfrac{a + j2\pi f}{b + j2\pi f}$, at the output, we obtain the following signal $y(t) = ae^{-bt}u(t)$.
   a. Find the transfer function $H(f)$ of the filter.
   b. Find the amplitude response of the filter.
   c. Find the phase response of the filter.
   d. Find the group delay of the filter.

e. Comment on the characteristic of the filter. Low-pass or high-pass?
f. Find the 3 dB bandwidth.
g. Estimate the essential bandwidth of the filter, if the essential band is required to contain 99% of the signal energy.
h. Find the impulse response.
i. Comment on the causality of the filter.

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ANALOG HABERLEŞME

UYGULAMA II

SORU 1)

a) $y(t) = \text{rect}\left(\frac{t-2T}{2T}\right) + \text{rect}\left(\frac{t+2T}{2T}\right)$

$Y(f) = 2T\,\text{sinc}(2fT)\,e^{-j2\pi f 2T} + 2T\,\text{sinc}(2fT)\,e^{j2\pi f 2T}$

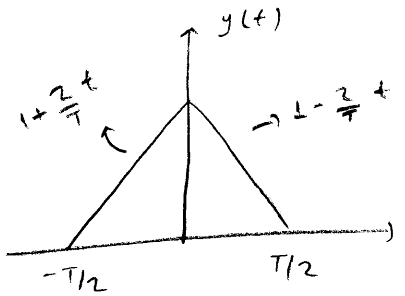
$Y(f) = 4T\,\text{Sinc}(2fT)\cos(4\pi f T)$

b) $y(t) = \text{rect}\left(\frac{t+T/2}{T}\right) - \text{rect}\left(\frac{t-T/2}{T}\right)$

$Y(f) = 2T\,\text{sinc}(fT)\,e^{j2\pi f T/2} - T\,\text{sinc}(fT)\,e^{-j2\pi f T/2}$

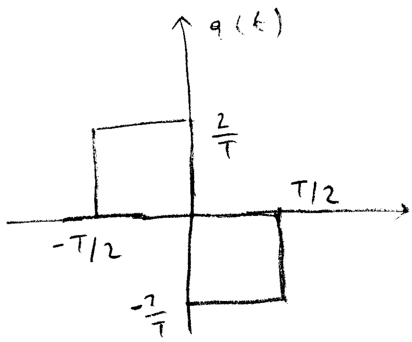
$Y(f) = 2jT\,\text{sinc}(fT)\sin(\pi f T)$

c)



$q(t) = \frac{d}{dt}\{y(t)\}$

$y(t) = \int_{-\infty}^{t} q(u)\,du$



$x(t) \longrightarrow X(f)$

$\frac{d}{dt}\{x(t)\} \rightarrow \frac{X(f)}{2j\pi f} + \cancel{\frac{1}{2}X(0)\,\delta(f)}^{\,0}$

$Y(f) = \frac{Q(f)}{j2\pi f}$ $\qquad Q(0) = 0$

$Q(f) = \frac{2}{T}\,2j\,\frac{T}{2}\cdot\text{Sinc}\left(\frac{fT}{2}\right)\cdot\sin\left(\frac{\pi f T}{2}\right)$

$Q(f) = 2j\,\text{sinc}\left(\frac{fT}{2}\right)\sin\left(\frac{\pi f T}{2}\right)$

$$Y(f) = \frac{1}{\pi f} \operatorname{sinc}\left(\frac{fT}{2}\right) \cdot \sin\left(\frac{\pi f T}{2}\right) \frac{T/2}{T/2}$$

$$Y(f) = \frac{T}{2} \cdot \operatorname{sinc}^2\left(\frac{fT}{2}\right)$$

SORU 2)

$$\xrightarrow{x(t)} \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = a \cdot e^{-bt} u(t)$$

$$Y(f) = \mathcal{F}\left\{ a e^{-bt} u(t) \right\} = a \int_0^\infty e^{-bt} \cdot e^{-2\pi f t} dt$$

$$= a \int_0^\infty e^{-(b+j2\pi f)t} dt$$

$$= \frac{a}{-(b+j2\pi f)} e^{-(b+j2\pi f)t} \Big|_0^\infty$$

$$= \frac{a}{(b+j2\pi f)}$$

a) $H(f) = ?$

$$Y(f) = X(f) \; H(f)$$

$$H(f) = \frac{Y(f)}{X(f)}$$

$$X(f) = \frac{a + j2\pi f}{b + j2\pi f}$$

$$H(f) = \frac{a}{a + j2\pi f} \rightarrow \text{transfer fonction}$$

b) $$H(f) = \frac{a}{a + j2\pi f} \cdot (a - j2\pi f) = \frac{a(a - j2\pi f)}{a^2 + (2\pi f)^2}$$

$$= \frac{a}{a^2 + (2\pi f)^2} \cdot (a - j2\pi f)$$

Reel   imajinel

$$|H(f)| = \frac{a}{a^2 + (2\pi f)} \sqrt{a^2 + (2\pi f)^2} = \frac{a}{\sqrt{a^2 + (2\pi f)^2}}$$

c) $$\theta(f) = \arctan\left(-\frac{2\pi f}{a}\right)$$

$$= -\arctan\left(\frac{2\pi f}{a}\right)$$

$$\tan^{-1}\theta = \arctan\left(\frac{Im}{Reel}\right)$$

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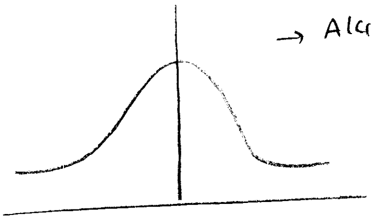
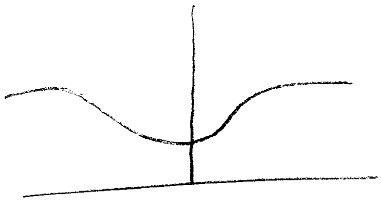
d) $\dfrac{d\theta}{dt} = \dfrac{-1}{1+\left(\dfrac{2\pi f}{a}\right)^2} \cdot \dfrac{2\pi}{a} = \dfrac{2\pi a}{a^2+(2\pi f)^2}$ $\qquad \arctan x = \dfrac{1}{1+x^2} \cdot x'$

e)  → Alçak geçiren

$H(0) = 1$
$f \to \infty \quad H(f) = 0$ } Alçak geçiren filtredir.

 → yüksek geçiren

f) $\dfrac{H(0)}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} = |H(f_{3dB})| \Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{a}{\sqrt{a^2+(2\pi f_{3dB})^2}}$

$\Rightarrow 2a^2 = a^2 + (2\pi f_{3dB})^2$

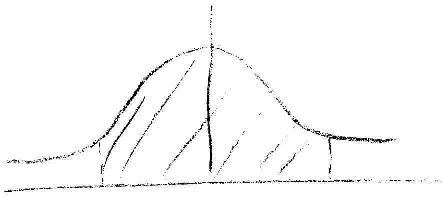
$$\boxed{f_{3dB} = \frac{a}{2\pi}}$$

g) $E_T = \displaystyle\int_{-\infty}^{\infty} |H(f)|^2 df = \int_{-\infty}^{\infty} \frac{a^2}{a^2+(2\pi f)^2} df$

$= \displaystyle\int_{-\infty}^{\infty} \frac{a^2}{a^2\left(1+\left(\frac{2\pi f}{a}\right)^2\right)} df$

$= \dfrac{a}{2\pi} \arctan\left(\dfrac{2\pi f}{a}\right)\Big|_{-\infty}^{\infty}$

$E_T = \dfrac{a}{2\pi}\left(\dfrac{\pi}{2}+\dfrac{\pi}{2}\right) = \dfrac{a}{2}$



$$0.99 \cdot \mathcal{E}_T = \int_{-B}^{B} |H(f)|^2 df$$

$$= \frac{a}{2\pi} \arctan\left(\frac{2\pi f}{a}\right)\Big|_{-B}^{B}$$

$$0.99 \cdot \frac{a}{2} = \frac{a}{2\pi}\left[\arctan\left(\frac{2\pi B}{a}\right) - \arctan\left(\frac{2\pi(-B)}{a}\right)\right]$$

$$0.99 = \frac{2}{\pi} \arctan\left(\frac{2\pi B}{a}\right)$$

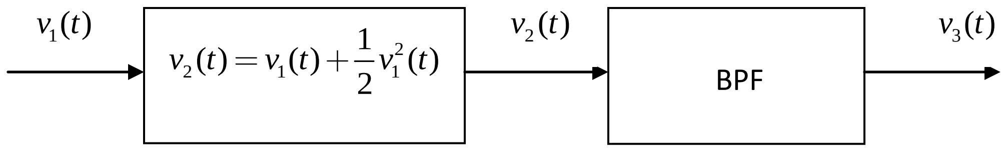
$$B = \frac{a}{2\pi} \tan\left(\frac{0.99\pi}{2}\right)$$

h) $H(f) \longrightarrow \mathcal{F}^{-1}\{H(f)\}$ $\qquad h(t) = a\, e^{-at} u(t)$

i) $t<0 \Rightarrow h(t)=0$ $\qquad$ Nedenseldir

ELM361 Analog Communication Systems
Homework #2
Due: 26.11.2013

1. Block diagram of a square law modulator is given as follows:



If the input signal $v_1(t) = c(t) + m(t)$ is the sum of the carrier wave $c(t) = \cos\ 8\pi f_m t$ and the message signal $m(t) = \cos\ 2\pi f_m t$,

a) Find and plot the spectrum of $v_2(t)$.
b) Determine the cutoff frequencies of the ideal band-pass filter, to extract the desired AM wave form $v_3(t)$. Plot the magnitude response of the filter.
c) Determine the modulation factor.
d) Calculate the carrier and the sideband powers.
e) Find the power efficiency of the modulation.

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Homework #2 - UYGULAMA III -

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Solution 1)

a) $V_2(t) = \left[m(t) + c(t) \right] + \frac{1}{2} \cdot \left[m(t) + c(t) \right]^2$

$= \left[\cos(2\pi f_m t) + \cos(8\pi f_m t) \right] + \frac{1}{2}\left[\cos(2\pi f_m t) + \cos(8\pi f_m t) \right]^2$

$= \cos(2\pi f_m t) + \cos(8\pi f_m t) + \frac{1}{2}\cos^2(2\pi f_m t) + \cos(2\pi f_m t) \cdot \cos(8\pi f_m t) + \frac{1}{2}\cos^2(8\pi f_m t)$

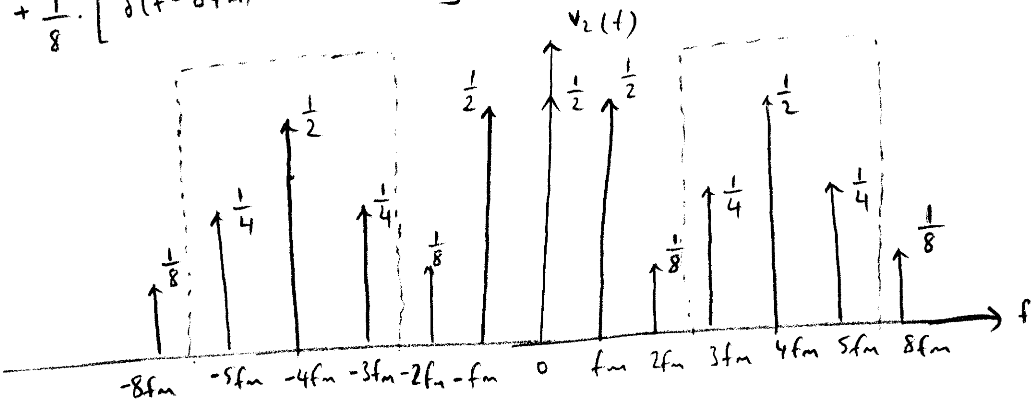
$= \cos(2\pi f_m t) + \cos(8\pi f_m t) + \frac{1}{4} \cdot \cos(4\pi f_m t) + \frac{1}{4}\cos(0) + \frac{1}{4}\cos(16\pi f_m t) + \frac{1}{4}\cos(0)$

$+ \frac{1}{2}\cos(10\pi f_m t) + \frac{1}{2}\cos(6\pi f_m t)$

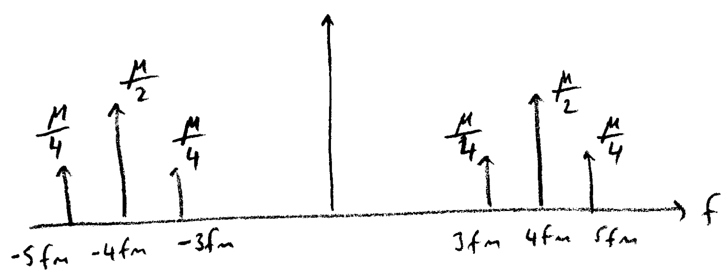
Hatırlatma!
$\cos(\theta_1) \cdot \cos(\theta_2)$
$= \frac{1}{2}\cos(\theta_1 + \theta_2) - \frac{1}{2} \cdot \cos(\theta_1 - \theta_2)$

$V_2(t) = \frac{1}{2} + \cos(2\pi f_m t) + \frac{1}{4}\cos(4\pi f_m t) + \frac{1}{2}\cos(6\pi f_m t)$
$+ \cos(8\pi f_m t) + \frac{1}{2}\cos(10\pi f_m t) + \frac{1}{4} \cdot \cos(16\pi f_m t)$

$V_2(f) = \frac{1}{2} \cdot \delta(f) + \frac{1}{2}\left[\delta(f - f_m) + \delta(f + f_m) \right] + \frac{1}{8} \cdot \left[\delta(f - 2f_m) + \delta(f + 2f_m) \right]$
$+ \frac{1}{4} \cdot \left[\delta(f - 3f_m) + \delta(f + 3f_m) \right] + \frac{1}{2}\left[\delta(f - 4f_m) + \delta(f + 4f_m) \right] + \frac{1}{4} \cdot \left[\delta(f - 5f_m) + \delta(f + 5f_m) \right]$
$+ \frac{1}{8} \cdot \left[\delta(f - 8f_m) + \delta(f + 8f_m) \right]$



AM modulation signal $= [1 + \mu \cdot \underbrace{\cos(2\pi f_m t)}_{m(t)}] \cdot \underbrace{\cos(8\pi f_m t)}_{c(t)}$



→ trigonometrik özdeşliği kullanıp çarp.

$\mu = 1$ bulunur.

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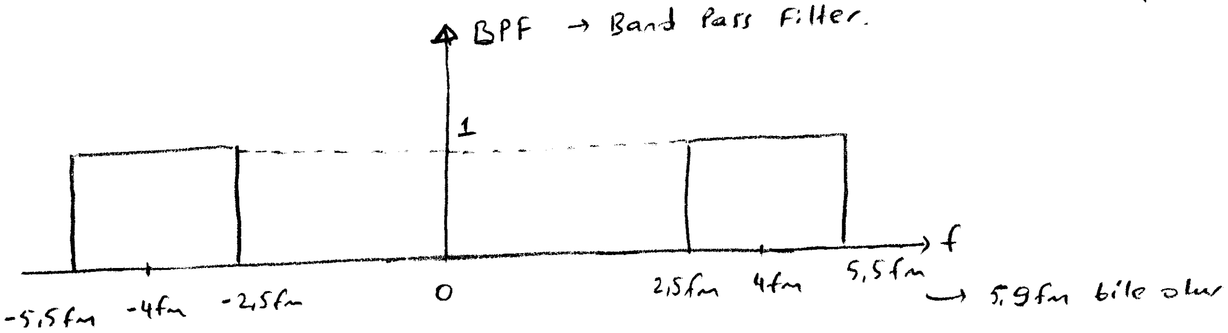

→ Bi sağda bi solda var. Ondan iki ile çarptık.

d) $P_c = 2.\left(\frac{1}{2}\right)^2 = \frac{1}{2}$ → carrier power

$P_s = 4.\left(\frac{1}{4}\right)^2 = \frac{1}{4}$ → sideband power.

e) power efficiency $\eta = \dfrac{P_s}{P_c + P_s} = \dfrac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{4}} = \dfrac{1}{3} \Rightarrow \%33$