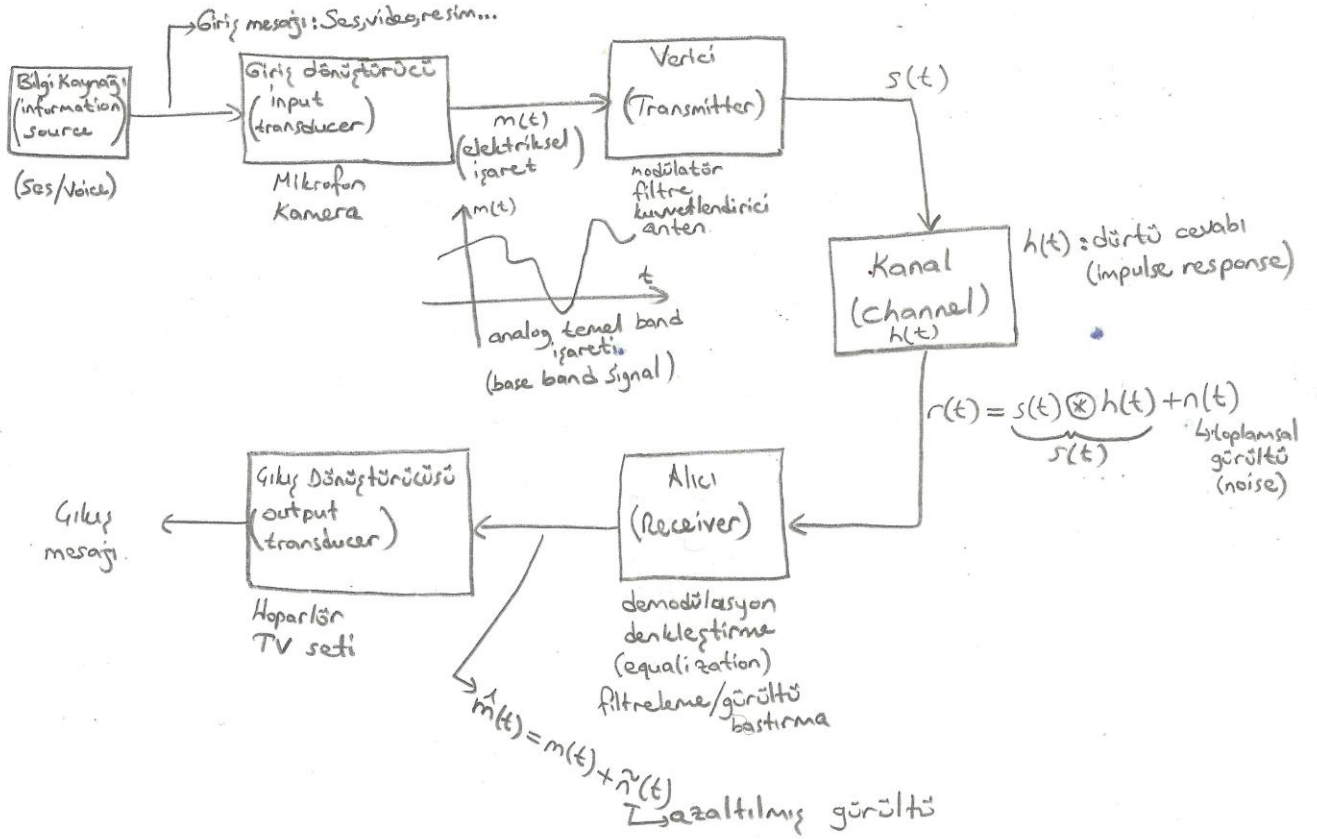


- Haberleşme: Bir sinyali bir veya birden fazla noktaya iletmeye denir.
- İşaret: Bilgi taşıyan fonksiyona denir.



### • Kanallar:

#### → 1) Kablolı kanallar (wireline channels)

- Twisted pair cable (Burulu çift kablo): Telefon haberleşmesinde.
- Coaxial cable (Eş eksenli kablo): Televizyon antenlerinde
- Fiber optic cable (Fiber kablo)
- Wave guide (Dalga kılavuzu)

#### → 2) Kablosuz/Telsiz Kanallar (wireless channels)

### • Bozucu etkiler:

- 1) Kanalin ideal olmaması → genlik ve faz bozulması olur. Çözüm: denkleştirme.
- 2) Eklenen gürültü (additive noise) →  $n(t)$
- 3) Zayıflama (attenuation)
- 4) Karışım (interference)
- 5) Sönümlenme (Fading)

• Önemli Kaynaklar (important resources):

Güç x Band Genişliği  
(Power) (Bandwidth)

• Mihenk Taşı (Figure of merit):

İşaretin (gücünün) gürültünün (gücüne) oranı: İGO  
(Signal-to-Noise Ratio: SNR)

$$\hat{m}(t) = m(t) + \tilde{n}(t)$$

İşaret    Gürültü

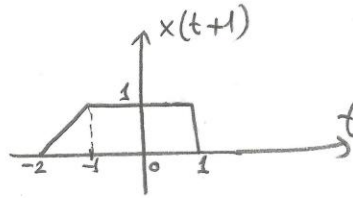
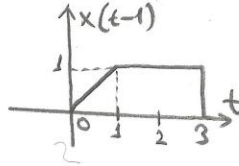
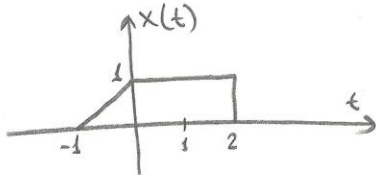
$$İGO = \frac{P_m}{P_n}$$

\* Bir haberleşme sisteminin amacı, gücü ve band genişliğini en verimli şekilde kullanarak İGO'yu maksimum yapmaktır.

• İşaretler ve Sistemler

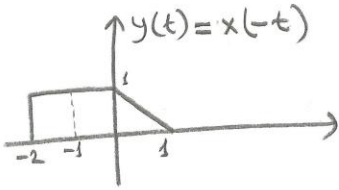
• İşlemler:

→ Zamanında öteleme (time-shifting)

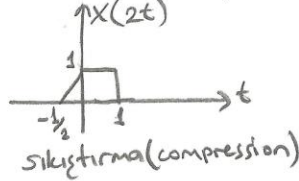
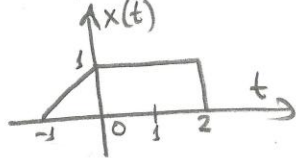


$x(t)$   
 $x(t+t_0)$   
 $t_0 < 0$  → gecikme  
 $t_0 > 0$  → ilerletme

→ Zamanın ters çevrilmesi (time reversal)



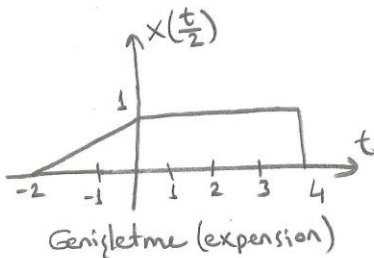
→ Zamanında ölçekleme (time scaling)



$$t \in [-1, 2]$$

$$2t \in [-1, 2]$$

$$t \in [-\frac{1}{2}, 1]$$



$$t \in [-1, 2]$$

$$\frac{t}{2} \in [-1, 2]$$

$$t \in [-2, 4]$$

$$x(t) \rightarrow x(at)$$

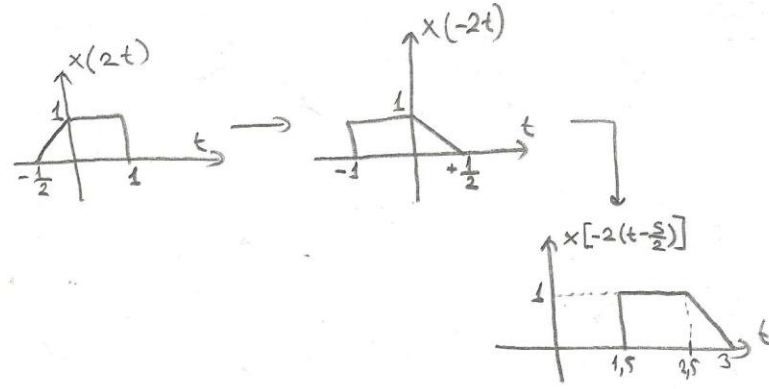
$a > 1$  → sıkıştırma

$0 < a < 1$  → genişletme

Örnek:

$$x(-2t+5) = x\left[-2\left(t-\frac{5}{2}\right)\right]$$

- 1-) Ölçekleme  $\rightarrow x(2t)$
- 2-) Zamanda ters çevirme  $\rightarrow x(-2t)$
- 3-) Öteleme  $\rightarrow x\left[-2\left(t-\frac{5}{2}\right)\right]$



$$t \in [-1, 2]$$

$$-2t+5 \in [-1, 2]$$

$$-2t \in [-6, -3]$$

$$t \in \left[\frac{3}{2}, 3\right]$$

### İşaretlerin Sınıflandırılması:

- $\rightarrow$  Sürekli zamanlı işaret  $- x(t)$ ,  $t \in \mathbb{R}$
- $\rightarrow$  Ayrık zamanlı işaret  $- x[n]$ ,  $n \in \mathbb{Z}$

- $\rightarrow$  Reel  $- x(t) \in \mathbb{R}$ ,  $x(t) = A \cos(2\pi f_c t + \theta)$
- $\rightarrow$  Kompleks  $- x(t) \in \mathbb{C}$ ,  $x(t) = A e^{j(2\pi f_c t + \theta)}$ ,  $e^{j\alpha} = \cos\alpha + j\sin\alpha$

$$x(t) = x_r(t) + jx_i(t) = |x(t)| e^{j\angle x(t)}$$

Genlik  $\rightarrow |x(t)| = \sqrt{x_r^2(t) + x_i^2(t)}$   
(Magnitude)

Faz  $\rightarrow \angle x(t) = \theta = -\arctan\left(\frac{x_i(t)}{x_r(t)}\right)$   
(Phase)

- $\rightarrow$  Deterministik/Belirgin  $\rightarrow$  herhangi bir  $t$  anında değeri bilinen (modellenebilen) işaret
- $\rightarrow$  Rasgele (Random)  $\rightarrow$  herhangi bir  $t$  anında değeri bilinmeyen (modellenemeyen) işaret

- $\rightarrow$  Periyodik/Periyodik olmayan  
 $\hookrightarrow \forall t \in \mathbb{R}, k \in \mathbb{Z} \quad x(t+kT_0) = x(t) \Rightarrow T_0 = \text{periyot}$

- $\rightarrow$  Nedensel/Nedensel olmayan  
(causal) (Noncausal)  
 $\hookrightarrow \forall t < 0, x(t) = 0$  ise  $x(t) \rightarrow$  nedensel

$\rightarrow$  Tek/Giift İşaretler

$\forall t, x(-t) = x(t)$  ise çift

$\forall t, x(-t) = -x(t)$  ise tek

$x(t)$  için;

$$x_{\text{çift}}(t) = \frac{x(t) + x(-t)}{2}$$

$$x_{\text{tek}}(t) = \frac{x(t) - x(-t)}{2}$$

$$x(t) = x_{\text{çift}}(t) + x_{\text{tek}}(t)$$

③

→ Bir işaretin enerjisi;

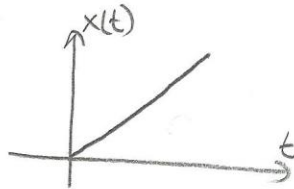
$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt \quad \bullet E_x < \infty \text{ ise } x(t), \text{ enerji işaretidir.}$$

→ Bir işaretin gücü;

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \quad \bullet P_x < \infty \text{ ise } x(t), \text{ güç işaretidir.}$$

•  $\sqrt{P_x}$ :  $x(t)$ 'nin RMS değeridir.

→  $x(t) = k \cdot t$   
↳ sbt



$x(t)$ , ne güç ne de enerji işaretidir.

→  $x(t)$  reel olsun;  $R = 1 \Omega$

$$E_x = \int_{-\infty}^{+\infty} \underbrace{\frac{x^2(t)}{R}}_{\text{Güç}} dt = \int_{-\infty}^{+\infty} \underbrace{x^2(t)}_{\text{Akım}} \cdot R dt$$

— Örnek:

$x(t) = A \cdot \cos(2\pi f_0 t + \theta)$ , Enerji/Güç işareti?

$$E_x = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} A^2 \cos^2(2\pi f_0 t + \theta) dt = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} \frac{A^2}{2} [1 + \cos(4\pi f_0 t + 2\theta)] dt$$

$$\bullet \cos^2 \alpha = \frac{1}{2} [1 + \cos(2\alpha)]$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2} \left[ t + \frac{\sin(4\pi f_0 t + 2\theta)}{4\pi f_0 t + 2\theta} \right]_{-T/2}^{T/2}$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2} \left[ T + \frac{\sin(2\pi f_0 T + 2\theta)}{2\pi f_0} - \frac{\sin(-2\pi f_0 T + 2\theta)}{-2\pi f_0} \right]$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2} \left[ T + \frac{\sin(2\pi f_0 T + 2\theta) + \sin(2\pi f_0 T + 2\theta)}{2\pi f_0} \right] \rightarrow \infty \Rightarrow \text{Enerji işareti değil}$$

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \frac{A^2}{2} [T + \dots] = \frac{A^2}{2} < \infty \Rightarrow \text{Güç işaretidir.} \checkmark$$



$$x(t) = \begin{cases} A \cos(2\pi f_0 t + \theta) \\ A \sin(2\pi f_0 t + \theta) \end{cases} \Rightarrow P_x = \frac{A^2}{2}$$

\* Periyodik işaretler güç işaretidir.

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \quad -\frac{T_0}{2} < T < \frac{T_0}{2}$$

+ Örnek:

$$x(t) = A \cos(2\pi f_0 t + \theta)$$

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A^2 \cos^2(2\pi f_0 t + \theta) dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \frac{A^2}{2} [1 + \cos(4\pi f_0 t + 2\theta)] dt$$

$$= \frac{A^2}{2T_0} \left[ \underbrace{\int_{-T_0/2}^{T_0/2} dt}_{T_0} + \int_{-T_0/2}^{T_0/2} \cos(4\pi f_0 t + 2\theta) dt \right] \quad \begin{matrix} f_0 \rightarrow T_0 = \frac{1}{f_0} \\ 2f_0 \rightarrow \frac{1}{2f_0} = \frac{T_0}{2} \end{matrix}$$

$$= \frac{A^2}{2}$$

27.02.2012

+ Örnek:

$$x(t) = C_1 \cos[2\pi f_1 t + \theta_1] + C_2 \cos[2\pi f_2 t + \theta_2], \quad f_1 \neq f_2$$

$$T_1 = \frac{1}{f_1} \quad T_2 = \frac{1}{f_2}$$

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left\{ C_1^2 \cos^2[2\pi f_1 t + \theta_1] + C_2^2 \cos^2[2\pi f_2 t + \theta_2] + 2C_1 C_2 \cos[2\pi f_1 t + \theta_1] \cos[2\pi f_2 t + \theta_2] \right\} dt$$

$$= \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \int_{-T/2}^{T/2} C_1^2 \cos^2[2\pi f_1 t + \theta_1] dt}_{\frac{C_1^2}{2}} + \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \int_{-T/2}^{T/2} C_2^2 \cos^2[2\pi f_2 t + \theta_2] dt}_{\frac{C_2^2}{2}} + \lim_{T \rightarrow \infty} \frac{2C_1 C_2}{T} \frac{1}{2} \left\{ \begin{matrix} \cos[2\pi(f_1 + f_2)t + \theta_1 + \theta_2] \\ \leftarrow \sim 0 \\ + \cos[2\pi(f_1 - f_2)t + \theta_1 - \theta_2] \\ \leftarrow \sim 0 \end{matrix} \right\} dt$$

$$P_x = \frac{C_1^2}{2} + \frac{C_2^2}{2}, \quad f_1 \neq f_2$$

En genel halde;

$$x(t) = x_1(t) + x_2(t) \Rightarrow P_x \neq P_{x_1} + P_{x_2}, f_1 \neq f_2$$

↳ manası: her zaman eşit olmayabilir.

Genelleştirme;

$$x(t) = \sum_{n=1}^{\infty} C_n \cos(2\pi f_n t + \theta_n), \forall i \neq j \text{ için } f_i \neq f_j \text{ olduğunda.}$$

$$P_x = \sum_{n=1}^{\infty} \frac{C_n^2}{2} = \frac{1}{2} \sum_{n=1}^{\infty} C_n^2$$

— Örnek:

$$x(t) = A e^{j2\pi f_0 t} \quad \cdot e^{j\alpha} = \cos \alpha + j \sin \alpha$$

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \underbrace{|A e^{j2\pi f_0 t}|^2}_{|A|^2} dt = \frac{|A|^2}{T_0} \int_{-T_0/2}^{T_0/2} dt = |A|^2$$

— Örnek:

$$x(t) = 5 \cos(2\pi 1000t) + 10 \sin(2\pi 2000t)$$

$$= 5^2 \cdot \frac{1}{2} + 10^2 \cdot \frac{1}{2} = \frac{125}{2}, f_1 \neq f_2$$

$f_1 = f_2 = 1000$  olsun;

$$x(t) = 5 \cos(2\pi 1000t) + 10 \cos(2\pi 1000t - 90)$$

$$= \frac{C_1^2}{2} + \frac{C_2^2}{2} = \frac{125}{2}$$

• Önemli İşaretler:

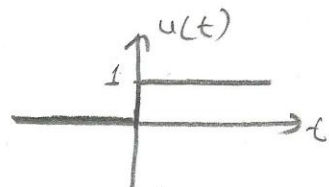
→ Sinüzoidal işaret.  $x(t) = A \cos(2\pi f_0 t + \theta)$

Genlik (Amplitude)  $\swarrow$   $\searrow$  frekans (Hz =  $\frac{1}{s}$ ) (Frequency)  $\nearrow$  faz (phase) (rad)

$\omega_0$ : açısal frekans (rad/s)

→ Kompleks üstel işaret.  $x(t) = A e^{j[2\pi f_0 t + \theta]}$

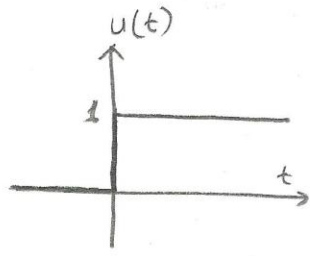
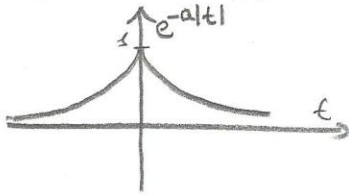
→ Birim basamak işareti.  $u(t) = \begin{cases} 1, t \geq 0 \\ 0, t < 0 \end{cases}$



$$a > 0, u(at) = u(t)$$

⑥

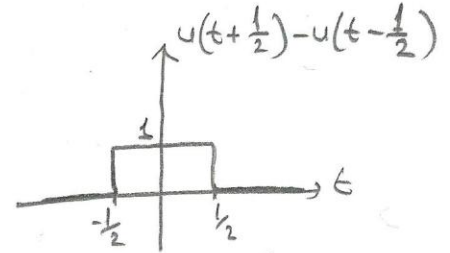
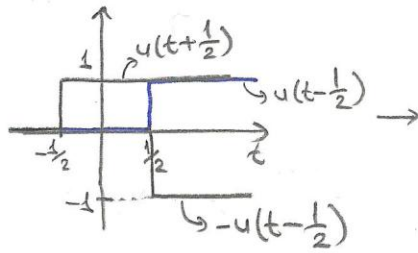
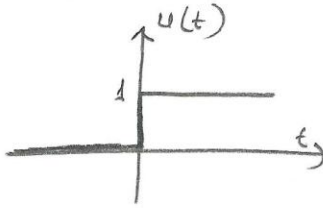
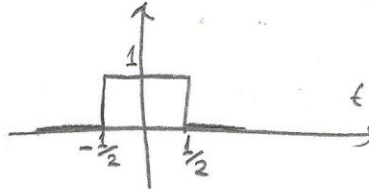
$$a > 0, e^{-a|t|} \cdot u(t)$$



$$e^{-a|t|} \cdot u(t) = \begin{cases} e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

→ • Dikdörtgen darbe işareti.  
(Rectangular pulse signal)

$$\pi(t) = \text{rect}(t) = \begin{cases} 1, & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0, & \text{diğer} \end{cases}$$

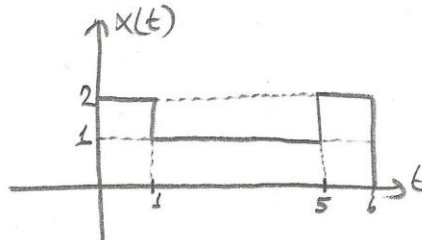
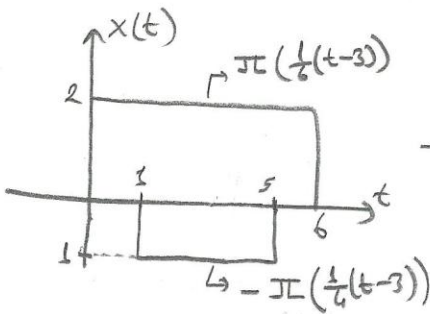
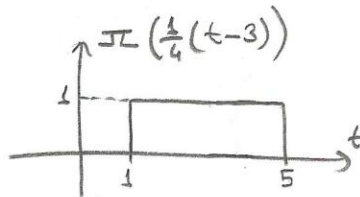
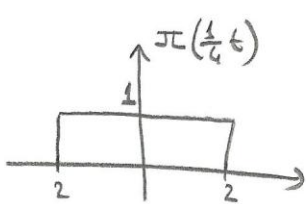
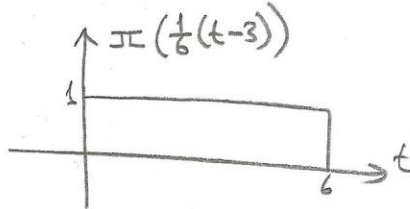
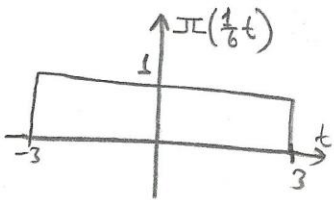


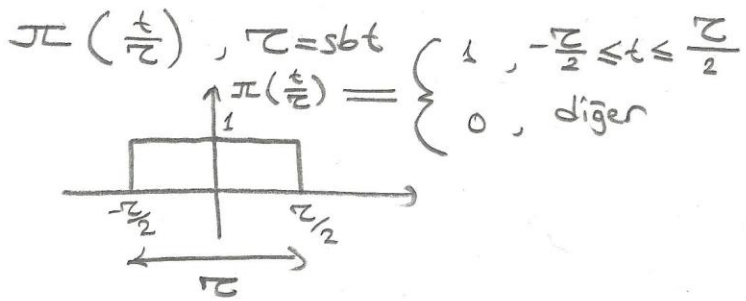
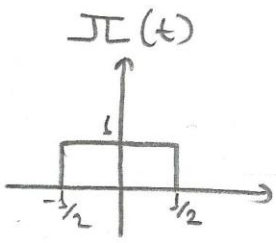
— + Örnek:

$$x(t) = 2 \pi\left(\frac{t-3}{6}\right) - \pi\left(\frac{t-3}{4}\right) \text{ qiziniz.}$$

$$\pi\left(\frac{1}{6}(t-3)\right)$$

$$\pi\left(\frac{1}{4}(t-3)\right)$$





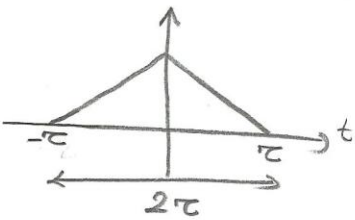
→ Üçgen darbe işareti.  
(Triangular pulse signal)

$$\Lambda(t) = \begin{cases} t+1, & -1 \leq t \leq 0 \\ -t+1, & 0 \leq t \leq 1 \\ 0, & \text{diğer} \end{cases}$$

$$\Lambda(t) = \Pi(t) * \Pi(t)$$

$$\Lambda\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \frac{|t|}{\tau}, & |t| \leq \tau \\ 0, & |t| \geq \tau \end{cases}$$

$$x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau) y(t-\tau) d\tau = \int_{-\infty}^{+\infty} x(\tau) y(\tau) d\tau$$



Konvolüsyon özelliği, konvolye edilen iki işaretin genişlikleri toplamı, sonucun genliğine eşittir.

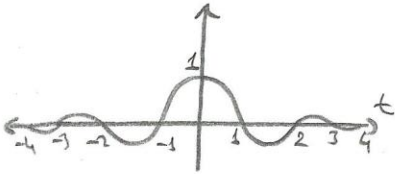
→ Sinc darbe işareti.

$$\text{sinc}(t) = \begin{cases} \frac{\sin(\pi t)}{\pi t}, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

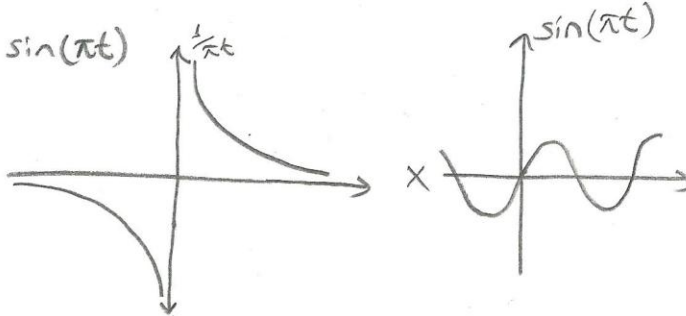
$$t = k \in \mathbb{Z} - \{0\} \Rightarrow \text{sinc}(t) = 0 \quad (k = \pm 1, 2, 3, \dots) \Rightarrow \text{sinc}(k) = 0$$

$$\text{sinc}(0) = \frac{\sin(0)}{0} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \lim_{t \rightarrow 0} \frac{\pi \cos(\pi t)}{\pi} = 1$$

$\text{sinc}(-t) = \text{sinc}(t) \rightarrow$  çift (even) fonksiyon

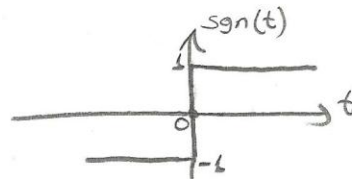


$$\text{sinc}(t) = \frac{1}{\pi t} \sin(\pi t)$$



→ İşaret fonksiyonu.  
(sign/signum signal)

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{cases}$$

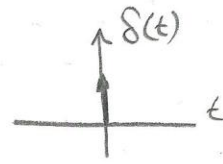


$$\text{sgn}(t) = u(t) - u(-t), \quad u(t) = \frac{1}{2}(1 + \text{sgn}(t))$$



→ Dürtü İşareti  
(Impulse/delta signal)

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & t \neq 0 \end{cases}$$



Distribution/Generalized Function

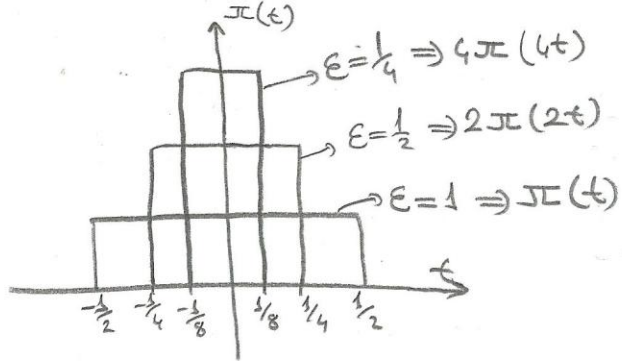
$$\int_{-\infty}^{+\infty} f(t) \cdot \delta(t) dt = f(0), \quad f(t), t=0 \text{ da sürekli olmak üzere (shifting property)}$$

$$f(t) = 1 \Rightarrow \int_{-\infty}^{+\infty} \delta(t) dt = 1$$

Alan = giddet = 1

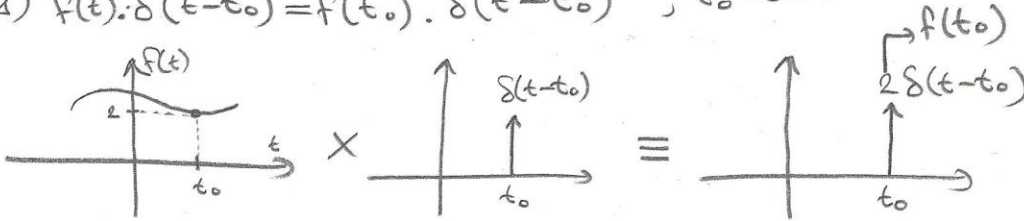
$$\delta(t) = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \text{sinc}\left(\frac{t}{\epsilon}\right)$$

$$\delta(t) = \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \pi\left(\frac{t}{\epsilon}\right)$$



• Özellikler:

→ 1)  $f(t) \cdot \delta(t-t_0) = f(t_0) \cdot \delta(t-t_0), \quad t_0 = sbt$



→ 2) Sifting (Sampling) Property

$$\int_a^b f(t) \delta(t-t_0) dt = f(t_0) \int_a^b \delta(t-t_0) dt = \begin{cases} f(t_0), & a < t_0 < b \\ 0, & \text{diğer} \end{cases}$$

$$f(t) = 1 \Rightarrow \int_a^b \delta(t-t_0) dt = \begin{cases} 1, & a < t_0 < b \\ 0, & \text{diğer} \end{cases}$$

→ 3)  $\forall a \neq 0 \quad \delta(at) = \frac{1}{|a|} \delta(t)$

$a = -1 \Rightarrow \delta(-t) = \delta(t) \rightarrow$  çift (even) fonksiyon.

→ 4)  $\delta(t)$  konvolüsyon işleminin birim elemanıdır.

$$f(t) * \delta(t) = f(t)$$

$$f(t) * \delta(t-t_0) = f(t-t_0)$$

$$f(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} f(\tau) \cdot \delta(\tau - t - t_0) d\tau = \int_{-\infty}^{\infty} f(\tau) \delta(\tau - (t-t_0)) d\tau = f(t-t_0)$$

⑨ Çift fonk. özelliği  
(- ile çarpıldı)

$$\rightarrow 5) \int_{-\infty}^t \delta(\tau) d\tau = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} = u(t) \quad \frac{du(t)}{dt} = \delta(t)$$

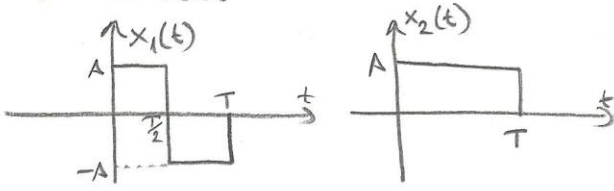
• Ortogonal ve Ortonormal İşaretler.

$\Psi_1(t), \Psi_2(t), \dots, \Psi_N(t)$  işaretleri  $[t_1, t_2]$  aralığında.

$$\int_{t_1}^{t_2} \Psi_m(t) \Psi_n^*(t) dt = \begin{cases} 0, & m \neq n \\ E_m, & m = n \end{cases} \text{ özelliğini sağlıyorsa ortogonal işaretlerdir.}$$

İlaveten  $\forall m$  için  $E_m = 1$  ise ortonormal işaretler olarak adlandırılır.

— Örnek:



$$\int_{-\infty}^{\infty} x_1(t) \cdot x_2^*(t) dt = \int_0^T x_1(t) \cdot x_2(t) dt$$

$$= \int_0^{T/2} A^2 dt + \int_{T/2}^T -A^2 dt = A^2 \frac{T}{2} - A^2 (T - \frac{T}{2}) = 0$$

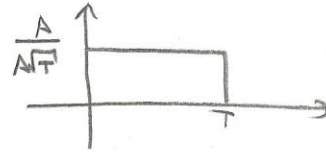
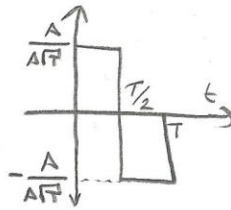
ortogonaldirler ✓

$$E_1 = \int_0^T |x_1(t)|^2 dt = \int_0^T A^2 dt = A^2 T$$

$$E_2 = A^2 T$$

$$x_1'(t) = \frac{x_1(t)}{\sqrt{E_1}} = \frac{x_1(t)}{A\sqrt{T}}$$

$$x_2'(t) = \frac{x_2(t)}{\sqrt{E_2}} = \frac{x_2(t)}{A\sqrt{T}}$$



$\left. \begin{matrix} E_{x_1'} = 1 \\ E_{x_2'} = 1 \end{matrix} \right\} x_1'(t) \text{ ve } x_2'(t) \text{ ortonormaldir}$

• Sistemler

$$x(t) \rightarrow \boxed{T[\cdot]} \rightarrow y(t) = T[x(t)]$$

$y(t) = \frac{d}{dt} x(t)$  Türev alıcı (sürekli)

$y[n] = x[n] - x[n-1]$  Türev alıcı (Ayrık zamanlı)

→ Lineer / Lineer olmayan sistemler

$$T[\underbrace{ax_1(t) + bx_2(t)}_{x_3(t)}] = y_3(t) = aT[x_1(t)] + bT[x_2(t)]$$

$$y(t) = \frac{d}{dt} x(t) \text{ lineer} \quad y(t) = \alpha x^2(t) \text{ lineer de\u011fil.}$$

$$y(t) = \int x(t) dt \text{ lineer} \quad y(t) = \beta \sqrt{x(t)} \text{ lineer de\u011fil.}$$

→ Zamanla de\u011fi\u015fen / Zamanla de\u011fi\u015meyen sistemler •  
(TV) (TIV)

$$\left. \begin{aligned} T[x(t)] &= y(t) \\ T[x(t-t_0)] &= y(t-t_0) \end{aligned} \right\} \text{TIV}$$

Sistemin giri\u015f-\u00e7ıkı\u015f ili\u015fkisi zamanla de\u011fi\u015miyorsa o sistem, TIV'dir.

+ \u00d6rnek:

$$y(t) = t x(t)$$

$$x_1(t) \rightarrow y_1(t) = t x_1(t)$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t) = t x_2(t) = t x_1(t-t_0) \neq \left\{ y_1(t-t_0) = (t-t_0) x_1(t-t_0) \right\}$$

sistem TV'dir.

+ \u00d6rnek:

$$y(t) = x(t+1) \rightarrow \text{TIV}$$

Lineer + Zamanla de\u011fi\u015meyen = L2D sistem (LTI)

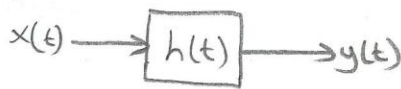
05.03.2012

→ Nedensel Sistem: Gelecekteki de\u011ferlerine ba\u011fı olmayıp, \u015imdiki ve ge\u00e7mi\u015fteki de\u011ferlerine ba\u011fı olan sistemlere denir.  
(Causal-Noncausal)

$$y(t_0) = T[x(t), t \leq t_0] \Rightarrow \text{Nedensel sistem}$$

→ L2D sistemin d\u00f6rt\u00f6 cevabı:  
(Impulse response)

$$y(t) = T[\delta(t)] = h(t)$$



$x(t)$  giri\u015fine  $\delta(t)$  uygulandığında alınan  $h(t)$  d\u00f6rt\u00f6 cevabı  $y(t)$ 'ye e\u015fittir.

$$y(t) = x(t) \otimes h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{+\infty} h(\tau) x(t-\tau) d\tau$$

+ \u00d6rnek:

$$y(t) = \int_{-\infty}^{+\infty} h(\tau) x(t-\tau) d\tau$$

$$t=0 \Rightarrow y(0) = \int_{-\infty}^{+\infty} h(\tau) x(-\tau) d\tau, \tau < 0 \text{ ilen } -\tau > 0 \text{ olur ve } x(-\tau) \text{ gelecekteki giri\u015f olur.}$$

Nedensellik \u015artı  $\left[ h(\tau) = 0, \tau < 0 \right]$  olmalıdır.

## • Genelleştirilmiş Fourier Serisi Açılımı •

$\Psi_1(t), \Psi_2(t), \dots, \Psi_N(t), \dots$  ortogonal işaretlerdir.

yaklaşık  $x(t) \rightarrow \hat{x}(t) = C_1 \Psi_1(t) + C_2 \Psi_2(t) + \dots + C_N \Psi_N(t) \quad t_1 \leq t \leq t_2$

→ Hata:

$$e(t) = x(t) - \hat{x}(t) \rightarrow \text{Hatânın enerjisi } E_e = \int_{t_1}^{t_2} |e(t)|^2 dt$$

$$C_n = \frac{1}{E_n} \int_{t_1}^{t_2} x(t) \Psi_n^*(t) dt; \quad n=1, 2, \dots, N \text{ seçilirse } E_e \text{ minimum olur.}$$

$\int_{t_1}^{t_2} x(t) \Psi_n^*(t) dt$  → Korelasyonu gösterir.

$x(t)$  işaret ailesi için  $N \rightarrow \infty$  gider iken  $E_e \rightarrow 0$ 'a gidiyorsa  $\{\Psi_n(t)\}_{n=1}^{\infty}$  ailesi tam (complete)dir. Bu durumda  $x(t)$  ailesi içindeki bütün işaretleri  $\{\Psi_n(t)\}_{n=1}^{\infty}$  ile ifade etmek mümkündür.

$$\left[ \begin{array}{l} x(t) \text{ 'nin genel} \\ \text{leştirilmiş four-} \\ \text{rier serisi açıl-} \\ \text{ımı} \end{array} \right] x(t) = \sum_{n=1}^{\infty} C_n \Psi_n(t), \quad t_1 \leq t \leq t_2$$

$\{\Psi_n(t)\}_{n=1}^{\infty}$  : Temel fonksiyonlar. (Basis functions)

$$C_n = \frac{1}{E_n} \int_{t_1}^{t_2} x(t) \Psi_n^*(t) dt; \quad n=1, 2, \dots, \infty$$

→ Parseval Teoremi:  $E_x = C_1^2 E_1 + C_2^2 E_2 + \dots = \sum_{n=1}^{\infty} C_n^2 E_n$ ,  $x(t)$ 'nin enerjisi işareti olduğu varsayıldı.

$\int_{t_1}^{t_2} \Psi_1^2(t) dt$

## • Trigonometrik Fourier Serisi Açılımı •

$\forall t, x(t+T_0) = x(t) \rightarrow$  periyodik

• FS açılımının mevcut olma şartları: (Kuvvetli Dirichlet şartları)

→ 1)  $\int_0^{T_0} |x(t)| dt < \infty$ , Zayıf Dirichlet şartı (Bu şart sağlanırsa FS açılımı var demektir.)  
Fakat her noktada yakınsamayabilir.

→ 2)  $x(t)$ 'nin bir periyotta sonlu sayıda maks, min ve süreksizlik noktası vardır.

Bu şartlar sağlanırsa  $x(t)$ 'nin FS açılımı vardır. Şartlar sağlanmasa da FS açılımı olabilir.



$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] , t_1 \leq t \leq t_1 + T_0 , \omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt , x(t) \text{'nin bir periyot boyunca integrali (DC bileşen)}$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(n\omega_0 t) dt ; n=1,2, \dots, \infty$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin(n\omega_0 t) dt ; n=1,2, \dots, \infty$$

$x(t)$  tek fonksiyon ise  $a_0, a_n \neq 0$   $a_n \rightarrow \text{tek} \times \text{çift} = \text{tek fonk.}$

$x(t)$  çift fonksiyon ise  $b_n = 0$   $b_n \rightarrow \text{çift} \times \text{tek} = \text{tek fonk}$

$$\forall n, m ; \int_{T_0} \sin(n\omega_0 t) \cos(m\omega_0 t) dt = 0 \rightarrow \sin(x) \cdot \cos(y) = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\forall n, m ; \int_{T_0} \cos(n\omega_0 t) \cos(m\omega_0 t) dt = \begin{cases} 0 , & n \neq m \\ \frac{T_0}{2} , & n = m \neq 0 \end{cases} \Rightarrow \cos^2(n\omega_0 t) = \frac{1}{2} [1 + \cos(2n\omega_0 t)]$$

$$\forall n, m ; \int_{T_0} \sin(n\omega_0 t) \sin(m\omega_0 t) dt = \begin{cases} 0 , & n \neq m \\ \frac{T_0}{2} , & n = m \neq 0 \end{cases} \Rightarrow \sin^2 \alpha = \frac{1}{2} (1 - \cos(2\alpha)) , \alpha = n\omega_0 t$$

$\rightarrow$  Parseval Teoremi:  $P_x = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} [a_n^2 + b_n^2] \rightarrow x(t)$  işaretinin gücünün açılımındaki katsayılar cinsinden ifadesi.

### Compact FS Expansion

$$a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) = C_n \cos(n\omega_0 t + \theta_n) , C_n = \sqrt{a_n^2 + b_n^2}$$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) , t_1 \leq t \leq t_1 + T_0$$

$$C_0 = a_0$$

$$\theta_n = -\arctan\left(\frac{b_n}{a_n}\right)$$

$\rightarrow$  Parseval Teoremi:  $P_x = C_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} C_n^2$

Fourier spektrumu

$C_n \times f = n \cdot f_0$  genlik spektrumu

$\theta_n \times f = n \cdot f_0$  faz spektrumu

Üstel FS Açılımı (Complex FS expansion)

$$x(t) = \sum_{n=-\infty}^{+\infty} x_n e^{jn\omega_0 t}$$

ortogonal işaretlerimiz  
(temel fonksiyon)

$$\int_{T_0} e^{jn\omega_0 t} \frac{(e^{jm\omega_0 t})^*}{e^{-jm\omega_0 t}} dt = \int_{T_0} e^{j(n-m)\omega_0 t} dt = \begin{cases} 0, & n \neq m \\ T_0, & n = m \end{cases}; e^{j\alpha} = \cos\alpha + j\sin\alpha$$

$$x_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

$\psi_n^*(t)$

→ Parseval Boremi:  $P_x = \sum_{n=-\infty}^{+\infty} |x_n|^2$

$$C_n \cos(n\omega_0 t + \theta_n) = C_n \frac{1}{2} [e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}] = \frac{1}{2} \underbrace{C_n e^{j\theta_n}}_{x_n} e^{jn\omega_0 t} + \frac{1}{2} \underbrace{C_n e^{-j\theta_n}}_{x_{-n}} e^{-jn\omega_0 t}$$

$$\left. \begin{aligned} x_n &= \frac{1}{2} C_n e^{j\theta_n} \\ x_{-n} &= \frac{1}{2} C_n e^{-j\theta_n} \end{aligned} \right\} n=1,2,\dots,\infty$$

$x_0 = C_0 = a_0$

Üstel FS spektrum:

$|x_n| \times f = n f_0$  Genlik spektrumu  
 $\theta_n \times f = n f_0$  Faz spektrumu

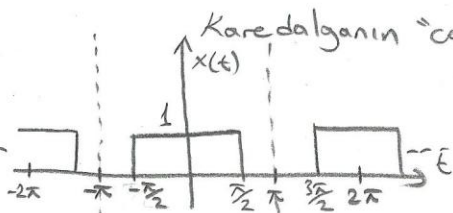
$$|x_n| = \frac{1}{2} C_n = |x_{-n}|$$

$$\angle x_n = \theta_n, \angle x_{-n} = -\theta_n$$

$$x_n = |x_n| e^{j\theta_n}$$

$$x_{-n} = |x_n| e^{-j\theta_n}$$

Örnek:



Karedalganın "compact" FS açılımını bulunuz.

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi} = 1; T_0 = 2\pi$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt = \frac{1}{T_0} \int_{-\pi/2}^{\pi/2} x(t) dt = \frac{1}{2\pi} \int_{-\pi}^{\pi} 1 dt = \frac{1}{2}$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(n\omega_0 t) dt = \frac{2}{2\pi} \int_{-\pi/2}^{\pi/2} 1 \cos(nt) dt = \frac{1}{\pi} \left[ \frac{\sin(nt)}{n} \right]_{-\pi/2}^{\pi/2} = \frac{1}{\pi n} \left[ \sin\left(\frac{n\pi}{2}\right) - \sin\left(-\frac{n\pi}{2}\right) \right]$$

$$= \frac{2}{\pi n} \sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0, & n = \text{çift} \\ -\frac{2}{\pi n}, & n = 3, 7, 11, 15, \dots \\ \frac{2}{\pi n}, & n = 1, 5, 9, 13, \dots \end{cases}$$

$b_n = 0$  olur çünkü  $x(t)$  çift fonksiyon.

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(t) - \frac{1}{3} \cos(3t) + \frac{1}{5} \cos(5t) - \frac{1}{7} \cos(7t) + \dots \right], \quad -\cos \alpha = \cos(\alpha - \pi)$$

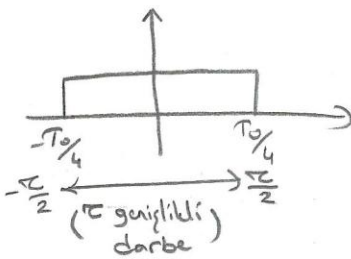
$a_0 = C_0$

$$= \frac{1}{2} + \frac{2}{\pi} \left[ \cos(t) + \frac{1}{3} \cos(3t - \pi) + \frac{1}{5} \cos(5t) + \frac{1}{7} \cos(7t - \pi) + \dots \right]$$

$C_0 = \frac{1}{2}$ ,  $C_n = \begin{cases} 0, & n = \text{çift} \\ \frac{2}{\pi n}, & n = \text{tek} \end{cases}$ ,  $\theta_n = \begin{cases} 0, & n \neq 3, 7, 11, 15, \dots \\ -\pi, & n = 3, 7, 11, 15, \dots \end{cases}$

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(\omega_0 t) + \frac{1}{3} \cos(3\omega_0 t - \pi) + \frac{1}{5} \cos(5\omega_0 t) + \frac{1}{7} \cos(7\omega_0 t - \pi) + \dots \right]$$

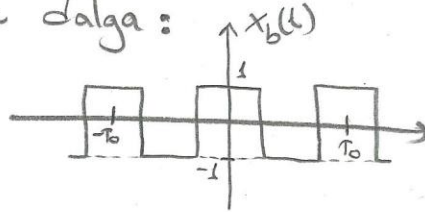
$a_0 = \frac{1}{2}$ ,  $a_n = \frac{2}{\pi n} \sin\left(n \frac{\pi}{2}\right)$ ,  $b_n = 0$



$a_0 = \frac{T_0}{T_0}$ ,  $a_n = \frac{2}{\pi n} \sin\left(n \omega_0 \frac{T_0}{2}\right)$ ,  $b_n = 0$ .

olsun.   
 ↖ genelleştirilmiş sonuç. ↗

Çift kutuplu kare dalga:



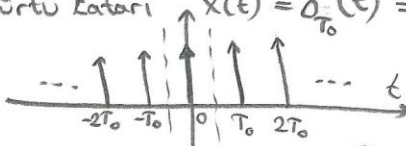
$x_b(t) = x(t) - x(t - T_0)$

$$x_b(t) = 2[x(t) - 0,5] = \frac{4}{\pi} \left[ \cos(\omega_0 t) + \frac{1}{3} \cos(3\omega_0 t - \pi) + \frac{1}{5} \cos(5\omega_0 t) + \frac{1}{7} \cos(7\omega_0 t - \pi) + \dots \right]$$

$a_0$ , DC elemanı olduğu için bu soru için yok olur.

Örnek:

Dürtü katarı  $x(t) = \delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$



a) "Compact" FS açılımı?

b) Üstel FS açılımı?

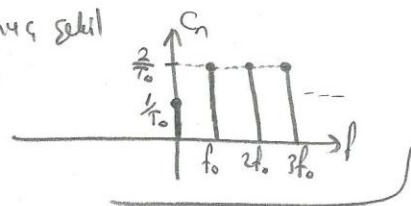
a)  $a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) dt = \frac{1}{T_0}$

$$\int_a^b f(t) \delta(t - t_0) = \begin{cases} f(t_0), & a < t_0 < b \\ 0, & \text{diğer} \end{cases}$$

$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \cos(n\omega_0 t) dt = \frac{2}{T_0}$

$\cos(n\omega_0 t) = 1$

\* Sonuç şekil



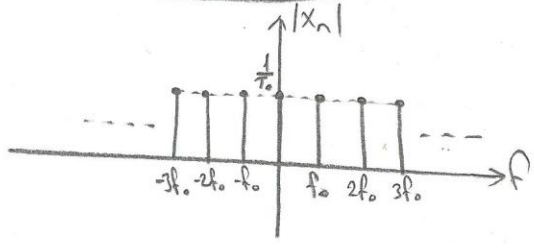
$b_n = 0$ ,  $x(t)$  çift fonk.

$x(t) = \frac{1}{T_0} + \frac{2}{T_0} \sum_{n=1}^{\infty} \cos(n\omega_0 t)$ ,  $C_0 = \frac{1}{T_0}$ ;  $C_n = \frac{2}{T_0}$   $\forall n$  için;  $\theta_n = 0$

b)  $x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t}$ ,  $x_n = \frac{1}{T_0} \int_{T_0} x(t) \underbrace{e^{-jn\omega_0 t}}_{(e^{jn\omega_0 t})^*} dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} e^{-jn\omega_0 t} dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} 1 dt = \frac{1}{T_0}$

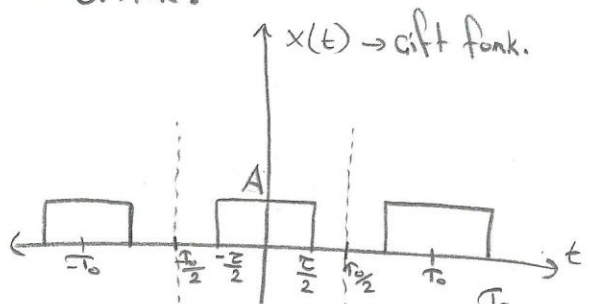
$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$

$x_n = \frac{1}{T_0}$   
 $|x_n| = \frac{1}{T_0}$   
 $\theta_n = 0$   
 $x_n = |x_n| e^{j\theta_n}$



$|x_n|$ : Genlik spektrumu  
 $|x_n| e^{j\theta_n}$ : B3y spektrumu

Örnek:



Üstel FS açılımı?  
 $\omega_0 = \frac{2\pi}{T_0}$

$x_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A e^{-jn\omega_0 t} dt = \frac{A}{T_0} \int_{-T_0/2}^{T_0/2} e^{-jn\omega_0 t} dt = \frac{A}{T_0} \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \Big|_{-T_0/2}^{T_0/2}$

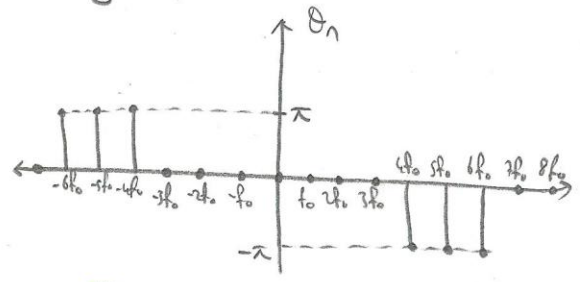
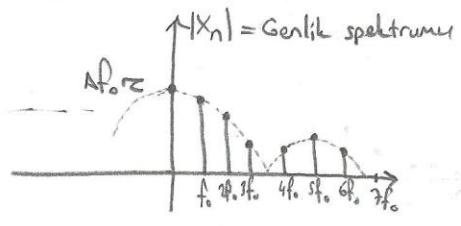
$= -\frac{A}{jn\omega_0 T_0} \left[ e^{-jn\omega_0 \frac{T_0}{2}} - e^{jn\omega_0 \frac{T_0}{2}} \right] = \frac{A}{2jn\pi} \left[ e^{jn\omega_0 \frac{T_0}{2}} - e^{-jn\omega_0 \frac{T_0}{2}} \right]$

$= \frac{A}{\pi n} \sin\left(n\omega_0 \frac{T_0}{2}\right) = \frac{A}{\pi n} \sin(\pi n f_0 \tau) = \frac{A}{\pi n} \frac{\sin(\pi n f_0 \tau)}{f_0 \tau} \times f_0 \tau = A f_0 \tau \frac{\sin(\pi n f_0 \tau)}{\pi n f_0 \tau}$

$= A f_0 \tau \text{sinc}(n f_0 \tau)$   $x_n = A f_0 \tau \text{sinc}(n f_0 \tau)$

$x(t) = A f_0 \tau \sum_{n=-\infty}^{\infty} \text{sinc}(n f_0 \tau) e^{jn\omega_0 t}$

$\frac{T_0}{4} < \tau < \frac{T_0}{3}$  olarak seçilirse;  $3f_0 < \frac{1}{\tau} < 4f_0$  olur.





# Fourier Dönüşümü

→ Periyodik olmayan işaretler için

→ Dirichlet Şartları

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$x(t) = \mathcal{F}\{X(f)\} = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$

$$X(f) = \underbrace{|X(f)|}_{\text{genelle spektrumu}} e^{j\theta_x(f)} \quad \text{faz spektrumu.}$$

— Örnek:

$$x(t) = \delta(t)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt = e^{-j2\pi ft} \Big|_{t=0} = 1$$

$$\boxed{\delta(t) \xrightarrow{\mathcal{F}} 1}$$

— Örnek:

$$X(f) = \delta(f)$$

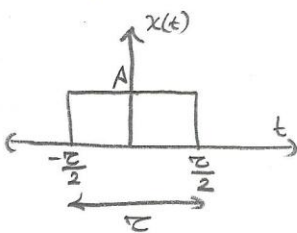
$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df = \int_{-\infty}^{\infty} \delta(f) e^{j2\pi ft} df = e^{j2\pi ft} \Big|_{f=0} = 1$$

$$\boxed{\delta(f) \xrightarrow{\mathcal{F}^{-1}} 1}$$

$$\boxed{1 \longleftrightarrow \delta(f)}$$

— Örnek:

$$x(t) = A \Pi\left(\frac{t}{\tau}\right)$$

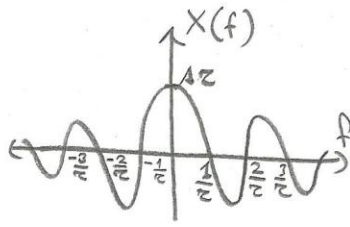


$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi ft} dt = A \left. \frac{e^{-j2\pi ft}}{-j2\pi f} \right|_{-\tau/2}^{\tau/2} \\ &= \frac{A}{-j2\pi f} \left[ e^{-j2\pi f \tau/2} - e^{j2\pi f \tau/2} \right] = \frac{A}{\pi f} \frac{e^{j\pi f \tau} - e^{-j\pi f \tau}}{j2} \\ &= \frac{A \tau}{\pi f \tau} \underbrace{\frac{\sin(\pi f \tau)}{\tau}}_{\text{sinc}} = A \tau \text{sinc}(f \tau) \end{aligned}$$

$$\boxed{x(t) = A \Pi\left(\frac{t}{\tau}\right) \xleftrightarrow{\mathcal{F}} X(f) = A \tau \text{sinc}(f \tau)}$$

$$\text{sinc}(k) = 0, k \in \mathbb{Z} - \{0\}$$

$$k = f\tau \Rightarrow f = \frac{k}{\tau}, k \in \mathbb{Z} - \{0\}$$



Yukarıdaki  $X(f)$  temel band işaretidir.  
En yüksek frekans sonsuza gittiği için  $BG = \infty$  olur.  
(Spektrumun sıfırdan farklı olduğu göz önünde bulundurulmalı)

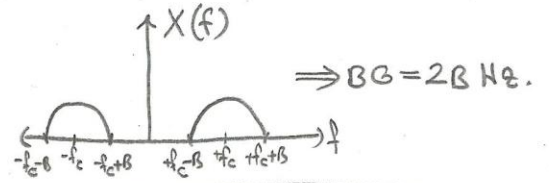
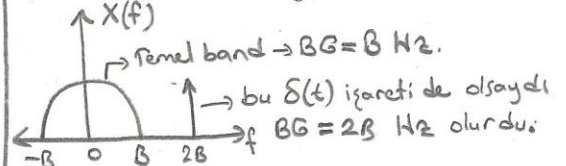
$$BG(\%) = \frac{1}{\tau}, (\text{Enerjinin } \%90'ı \text{ } -\frac{1}{\tau} \text{ ile } \frac{1}{\tau} \text{ arasındadır.})$$

$$BG = \frac{1}{\tau} \times \text{Zamandaki genişlik} = \tau$$

↳ Ters orantı.

• Band Geniliği (BG) Bandwidth:

- Temel band (Baseband) işaretleri için en yüksek pozitif frekans.
- Band geçiren (Bandpass) işaretler için spektrumunun sıfırdan farklı olduğu pozitif frekanslar bölgesidir.



→ Fourier Dönüşümünün Özellikleri:

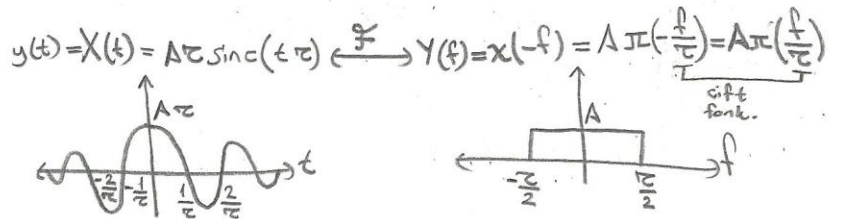
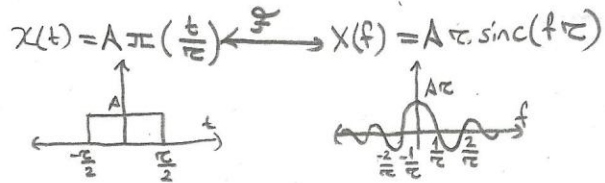
→ Lineerlik:  $a_1, a_2$  skalar  
 $x_1(t), x_2(t)$   
 $x_1(t) \xrightarrow{F} X_1(f)$   
 $x_2(t) \xrightarrow{F} X_2(f)$

$$\underbrace{a_1 x_1(t) + a_2 x_2(t)}_{x_3(t)} \xrightarrow{F} \underbrace{a_1 X_1(f) + a_2 X_2(f)}_{X_3(f)}$$

→ Dualite (Simetri) özelliği:

$$x(t) \xrightarrow{F} X(f)$$

$$y(t) = X(t) \xrightarrow{F} Y(f) = x(-f)$$



$$A\tau \text{sinc}(t\tau) \xrightarrow{F} A\tau \text{rect}\left(\frac{f}{\tau}\right)$$

→ Zamanda Öteleme Özelliği:

$$x(t) \xrightarrow{F} X(f)$$

$$y(t) = x(t - t_0) \xrightarrow{F} X(f) e^{-j2\pi f t_0} = Y(f), t_0 = \text{sbb.}$$

$|Y(f)| = |X(f)|$ , faz spektrumu etkilenir yalnızca. Genlik spektrumu değişmez.  
genlik spektrumu

İspat:  $Y(f) = \int_{-\infty}^{\infty} y(t) e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} x(t-t_0) e^{j2\pi ft} dt = \int_{-\infty}^{\infty} x(\alpha) e^{-j2\pi f(\alpha+t_0)} d\alpha = e^{-j2\pi ft_0} \int_{-\infty}^{\infty} x(\alpha) e^{j2\pi f\alpha} d\alpha = e^{-j2\pi ft_0} X(f)$

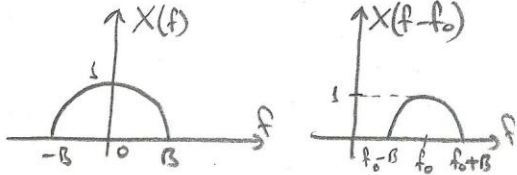
$t-t_0 = \alpha$   
 $t = \alpha+t_0$   
 $dt = d\alpha$

→ Frekansla öteleme özelliği:

$x(t) \xleftrightarrow{\mathcal{F}} X(f)$

$x(t) e^{j2\pi f_0 t} \xleftrightarrow{\mathcal{F}} X(f-f_0)$   
 $f_0 = sbt.$

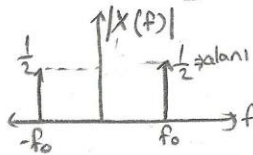
İspat:  $\mathcal{F}\{x(t) e^{j2\pi f_0 t}\} = \int_{-\infty}^{\infty} x(t) e^{j2\pi f_0 t} e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} x(t) e^{-j2\pi(f-f_0)t} dt = X(f-f_0)$



Örnek:  $x(t) = \cos(2\pi f_0 t) = \frac{1}{2} [e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}] \xleftrightarrow{\mathcal{F}} X(f)$

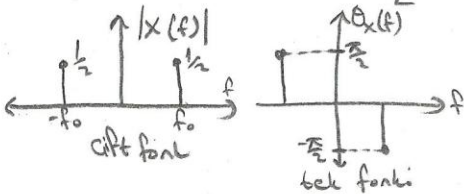
$X(f) = \frac{1}{2} [\delta(f-f_0) + \delta(f+f_0)]$

$1 \xleftrightarrow{\mathcal{F}} \delta(t)$   
 $x(t) e^{j2\pi f_0 t} \xleftrightarrow{\mathcal{F}} X(f-f_0)$   
 $e^{j2\pi f_0 t} \xleftrightarrow{\mathcal{F}} \delta(f-f_0)$



Örnek:  $x(t) = \sin(2\pi f_0 t) = \frac{1}{2j} [e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}] = \frac{1}{2} [e^{j2\pi f_0 t} e^{-j\frac{\pi}{2}} + e^{-j2\pi f_0 t} e^{j\frac{\pi}{2}}] \cdot \left[ \begin{matrix} j = e^{j\frac{\pi}{2}} \\ \frac{1}{j} = e^{-j\frac{\pi}{2}} \end{matrix} \right]$

$x(t) \xleftrightarrow{\mathcal{F}} X(f) = \frac{1}{2} [\delta(f-f_0) e^{-j\frac{\pi}{2}} + \delta(f+f_0) e^{j\frac{\pi}{2}}]$



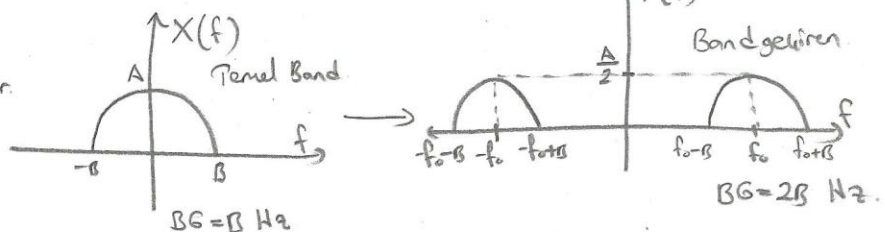
$\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2} [\delta(f-f_0) + \delta(f+f_0)]$   
 $\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2} [\delta(f-f_0) e^{j\frac{\pi}{2}} + \delta(f+f_0) e^{-j\frac{\pi}{2}}]$

→ Modülasyon Özelliği:

$x(t) \xleftrightarrow{\mathcal{F}} X(f)$

$y(t) = x(t) \cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} Y(f) = \frac{1}{2} [X(f-f_0) + X(f+f_0)]$

(modüle eden) işaret ( taşıyıcı) işaret  
 $x(t), \cos(2\pi f_0 t)$ 'nin şebekini değiştirir.





→ Ölçekleme özelliği: (scaling)

$$x(t) \xrightarrow{\mathcal{F}} X(f)$$

$$x(at) \xrightarrow{\mathcal{F}} \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$x(2t) \xrightarrow{\mathcal{F}} \frac{1}{2} X\left(\frac{f}{2}\right) \rightarrow \text{işaretin zamandaki genişliği ile Band Genişliği ters orantılıdır.}$$

→ Konvolüsyon özelliği:

$$x(t) \otimes y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$$

$$z(t) = x(t) \otimes y(t) \xrightarrow{\mathcal{F}} Z(f) = X(f) \cdot Y(f) \quad ; \quad \left[ \text{genişlik}\{z(t)\} = \text{genişlik}\{x(t)\} + \text{genişlik}\{y(t)\} \right]$$

$$z(t) = x(t) \cdot y(t) \xrightarrow{\mathcal{F}} Z(f) = X(f) \otimes Y(f)$$

$$BG_{top} = BG_1 + BG_2$$

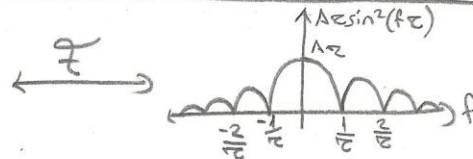
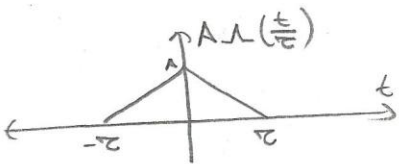
— Örnek:

$$\Lambda(t) = \Pi(t) \otimes \Pi(t)$$

$$\mathcal{F}\{\Lambda(t)\} = \underbrace{\mathcal{F}\{\Pi(t)\}}_{\text{sinc}(f)} \cdot \underbrace{\mathcal{F}\{\Pi(t)\}}_{\text{sinc}(f)} = \text{sinc}^2(f)$$

$$\mathcal{F}\left\{A \Lambda\left(\frac{t}{\tau}\right)\right\} = A \tau \text{sinc}^2(f \tau)$$

$$A \Lambda\left(\frac{t}{\tau}\right) \xrightarrow{\mathcal{F}} A \tau \text{sinc}^2(f \tau)$$



→ Integral özelliği:

$$x(t) \xrightarrow{\mathcal{F}} X(f)$$

$$y(t) = \int_{-\infty}^t x(\tau) d\tau \xrightarrow{\mathcal{F}} Y(f) = \frac{X(f)}{j2\pi f} + \frac{1}{2} X(0) \delta(f)$$

↳ frekans arttıkça Y(f) ifareti sönümlenir. (Alçak geçiren filtre gibi kalır.)

— Örnek:

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau \xrightarrow{\mathcal{F}} \mathcal{F}\{u(t)\} = \frac{1}{j2\pi f} + \frac{1}{2} \delta(f)$$

$$\mathcal{F}\{u(t)\} = \frac{1}{j2\pi f} + \frac{1}{2} \delta(f)$$

— Örnek:

$$\text{sgn}(t) = u(t) - u(-t) \xrightarrow{\mathcal{F}} \mathcal{F}\{\text{sgn}(t)\} = \left\{ \frac{1}{j2\pi f} + \frac{1}{2} \delta(f) \right\} - \frac{1}{|-1|} \left\{ \frac{1}{j2\pi f} + \frac{1}{2} \delta(f) \right\}$$

$$\mathcal{F}\{\text{sgn}(t)\} = \frac{1}{j2\pi f}$$

cift fark  
 $\delta(-f) = \delta(f)$

$$\begin{aligned} u(t) &\xrightarrow{\mathcal{F}} \frac{1}{j2\pi f} + \frac{1}{2} \delta(f) \\ \text{sgn}(t) &\xrightarrow{\mathcal{F}} \frac{1}{j2\pi f} \end{aligned}$$



→ Türev Özelliği:

$$x(t) \xrightarrow{\mathcal{F}} X(f)$$

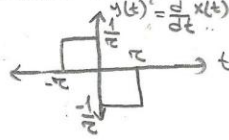
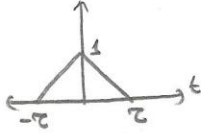
$$y(t) = \frac{d}{dt} x(t) \xrightarrow{\mathcal{F}} Y(f) = j2\pi f X(f)$$

↳ frekans arttıkça  $Y(f)$  ifareti büyür. (Yüksel geçiren filtre)

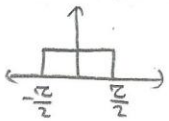
$$\frac{d^n x(t)}{dt^n} \xrightarrow{\mathcal{F}} (j2\pi f)^n X(f)$$

Örnek:

$x(t) = \Lambda\left(\frac{t}{\tau}\right)$ 'nin Fourier Dönüşümünü türev özelliğini kullanarak bulunuz.



$$z(t) = A \text{rect}\left(\frac{t}{\tau}\right) \xrightarrow{\mathcal{F}} Z(f) = A \tau \text{sinc}(f\tau) = \text{sinc}(f\tau)$$



$$y(t) = z\left(t + \frac{\tau}{2}\right) - z\left(t - \frac{\tau}{2}\right) \xrightarrow{\mathcal{F}} Y(f) = Z(f) e^{j2\pi f \frac{\tau}{2}} - Z(f) e^{-j2\pi f \frac{\tau}{2}}$$

$$Y(f) = Z(f) [e^{j\pi f \tau} - e^{-j\pi f \tau}]$$

$$Y(f) = \text{sinc}(f\tau) [e^{j\pi f \tau} - e^{-j\pi f \tau}]$$

$$y(t) = \frac{d}{dt} x(t) \Rightarrow Y(f) = j2\pi f X(f) \Rightarrow X(f) = \frac{Y(f)}{j2\pi f}$$

$$X(f) = \text{sinc}(f\tau) \frac{[e^{j\pi f \tau} - e^{-j\pi f \tau}]}{j2\pi f} = \text{sinc}(f\tau) \frac{\sin(\pi f \tau)}{\pi f \tau} \tau = \tau \text{sinc}^2(f\tau)$$

• Periyodik İşaretlerin Fourier Dönüşümü:

$$\forall t \quad x(t + kT_0) = x(t) ; k \in \mathbb{Z}$$

$$x(t) = \sum_{n=-\infty}^{+\infty} x_n e^{j2\pi n f_0 t} \xrightarrow{\mathcal{F}} X(f) = \sum_{n=-\infty}^{+\infty} x_n \delta(f - n f_0)$$

$$; \omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

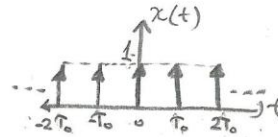
$$; x_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-j2\pi n f_0 t} dt$$

$$; e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} \delta(f - f_0)$$

$$; e^{-j2\pi f_0 t} \xrightarrow{\mathcal{F}} \delta(f + f_0)$$

Örnek:

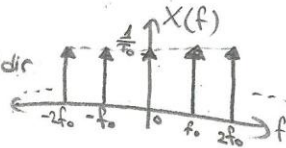
$$x(t) = \delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$



$$x_n = \frac{1}{T_0} \rightarrow x(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{j2\pi n f_0 t} \xrightarrow{\mathcal{F}} X(f) = f_0 \sum_{n=-\infty}^{\infty} \delta(f - n f_0)$$

Dürtü Katarı;

Fourier dönüşümü kendisine benzeyen nadir işaretlerden biridir



→ Rayleigh Teoremi:

$$\int_{-\infty}^{\infty} x(t) y^*(t) dt = \int_{-\infty}^{\infty} X(f) Y^*(f) df \quad \xrightarrow{x(t)=y(t) \text{ iken}} \text{Parseval teoremini verir}$$

$$\underbrace{\int_{-\infty}^{\infty} |x(t)|^2 dt}_{E_x} = \int_{-\infty}^{\infty} \underbrace{|X(f)|^2 df}_{\text{enerji yoğunluk fonksiyonu}} E_x$$

⚡ Zaman uzayındaki enerji frekans uzayındaki enerjiye eşittir

$x(t)$ 'nin bulunması zor ise  $X(f)$ 'i bulunarak sonucu gidebilir.

$x(t) = A \pi \text{sinc}(f\tau)$ 'nin bulunması zordur. →  $X(f) = A \text{rect}(f\tau)$ 'su ile devam edilebilir.

$x(t)$  → reel bir işaret ise ⇒  $X(-f) = X^*(f)$

$x(t)$  → reel ve çift bir işaret ise ⇒  $X(f)$  da reel ve çifttir. →  $\begin{cases} X^*(f) = X(f) \\ X(-f) = X(f) \end{cases}$

$x(t)$  → reel ve tek bir işaret ise ⇒  $X(f)$  sanal ve tektir. →  $\begin{cases} X^*(f) = -X(f) \\ X(-f) = -X(f) \end{cases}$

— Örnek:

$x(t)$  reel ve çift;  $X(f)$ 'nin reel ve çift olduğunu gösteriniz.

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{j2\pi ft} dt$$

$$X^*(f) = \int_{-\infty}^{\infty} \underbrace{x^*(t)}_{\substack{=x(t) \\ \text{(reel)}}} e^{j2\pi ft} dt = \int_{-\infty}^{\infty} \underbrace{x(-t)}_{\substack{\text{(çift)} \\ -t=t' \\ -dt=dt'}} e^{j2\pi ft} dt = - \int_{-\infty}^{\infty} x(t') e^{-j2\pi ft'} dt' = \int_{-\infty}^{\infty} x(t') e^{j2\pi ft'} dt' = X(f)$$

$X^*(f) = X(f) \rightarrow \text{reel} \checkmark$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$X(-f) = \int_{-\infty}^{\infty} x(t) e^{j2\pi ft} dt = \dots = X(f)$$

$X(-f) = X(f)$  çift fonk. ✓✓

→ L2D sistemlerden işaret iletilmesi:

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) \otimes h(t)$

$X(f) \quad H(f) \quad Y(f) = X(f) \cdot H(f)$

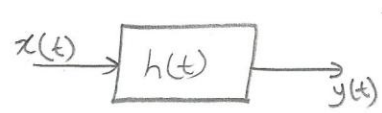
transfer fonksiyonu

$$Y(f) = |Y(f)| e^{j\theta_Y(f)} = |X(f)| e^{j\theta_X(f)} \cdot \underbrace{|H(f)|}_{\text{genil cevabi}} e^{j\theta_H(f)} \quad \text{faz cevabi.}$$

$$|Y(f)| = |X(f)| \cdot |H(f)|$$

$$\theta_Y(f) = \theta_X(f) + \theta_H(f)$$

Örnek:



$$x(t) = 3 + 5 \cos(3t + \frac{\pi}{4})$$

$$H(f) = j20\pi f$$

- a)  $x(t)$ 'nin periyodu nedir?  $y(t)$ 'nin periyodu nedir?
- b)  $P_x = ?$
- c)  $y(t) = ?$
- d)  $P_y = ?$

a)  $x(t) = 3 + 5 \cos(3t + \frac{\pi}{4})$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} = 3 \quad T_0 = \frac{2\pi}{3}$$

$\Rightarrow y(t)$ 'nin periyodu  $= \frac{2\pi}{3}$  [LTD sistem olduğu için]

b)  $P_x = 3^2 + 5^2 \cdot \frac{1}{2} = 21,5$  Watt

c)  $y(t) = ?$

$$H(f) = j20\pi f \xrightarrow{\mathcal{F}^{-1}} h(t) = 10 \frac{d}{dt}(\cdot) \quad [j2\pi f, \text{ zamanla "türev" demektir}]$$

$$Y(f) = H(f) \cdot X(f) ; X(f) = 3\delta(f) + \frac{5}{2} [\delta(f-f_0)e^{j\frac{\pi}{4}} + \delta(f+f_0)e^{-j\frac{\pi}{4}}]$$

$$= (j20\pi f) \cdot X(f)$$

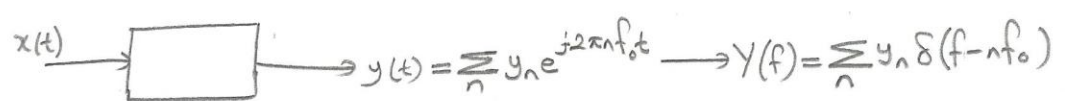
$$= j20\pi f \cdot 3\delta(f) + j20\pi f_0 \cdot \frac{5}{2} \delta(f-f_0)e^{j\frac{\pi}{4}} + j20\pi(-f_0) \cdot \frac{5}{2} \delta(f+f_0)e^{-j\frac{\pi}{4}}$$

$$= 75j \delta(f-f_0)e^{j\frac{\pi}{4}} - 75j \delta(f+f_0)e^{-j\frac{\pi}{4}} \quad \bullet [j = e^{j\frac{\pi}{2}}]$$

$$Y(f) = 75 [\delta(f-f_0)e^{j\frac{3\pi}{4}} + \delta(f+f_0)e^{-j\frac{3\pi}{4}}] \cdot \frac{2}{2}$$

$$y(t) = \mathcal{F}^{-1}\{Y(f)\} = 150 \cos(\underbrace{2\pi f_0 t}_{\omega=3} + \frac{3\pi}{4}) = 150 \cos(3t + \frac{3\pi}{4})$$

d)  $P_y = 150^2 \cdot \frac{1}{2}$  Watt



$$x(t) = \sum_n x_n e^{j2\pi n f_0 t} \longrightarrow X(f) = \sum_n x_n \delta(f - n f_0)$$

$$Y(f) = H(f) \cdot X(f) = H(f) \cdot \sum_n x_n \delta(f - n f_0) = \sum_n \underbrace{x_n H(n f_0)}_{= y_n} \delta(f - n f_0)$$

$$y(t) = \mathcal{F}^{-1}\{Y(f)\} = \sum_n \underbrace{x_n H(n f_0)}_{y_n} e^{j2\pi n f_0 t}$$

• Enerji ve Enerji Yoğunluğu Fonksiyonu (EYF)  
(Energy spectral density - ESD)

→ Enerji:  $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

→ Parseval Teoremi:  $E_x = \int_{-\infty}^{\infty} \underbrace{x(t) x^*(t)}_{|x(t)|^2} dt = \int_{-\infty}^{\infty} x(t) \left[ \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \right] dt$

$$= \int_{-\infty}^{\infty} x(t) \int_{-\infty}^{\infty} X^*(f) e^{-j2\pi f t} df dt$$

$$= \int_{-\infty}^{\infty} X^*(f) \left[ \int_{-\infty}^{\infty} x(t) e^{j2\pi f t} dt \right] df = \int_{-\infty}^{\infty} \underbrace{|X(f)|^2}_{\substack{\text{Enerji yoğunluğu fonksiyonu. (EYF)} \\ E_x(f)}} df$$

$E_x(f) = |X(f)|^2 \rightarrow$  pozitif, reel ve çift

$x(t) \rightarrow E_x(f)$   
(volt)  $\rightarrow$   $\left(\frac{\text{volt}^2}{\text{Hz}}\right)$

• Özlülük Fonksiyonu ve EYF  
(Autocorrelation)

$R_x(\tau) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt = \int_{-\infty}^{\infty} x(t) \underbrace{x^*(t+\tau)}_{\text{çift fonk.}} dt = \int_{-\infty}^{\infty} x^*(t) x(t-\tau) dt$

$R_x(-\tau) = R_x(\tau)$

$R_x(\tau) = x(\tau) \otimes x^*(-\tau) \xleftrightarrow{\mathcal{F}} \mathcal{F}\{R_x(\tau)\} = |X(f)|^2 = E_x(f)$

→ /spat:

$x(t)$  reel olsun.

$R_x(\tau) = \int_{-\infty}^{\infty} x(t) x(t-\tau) dt = \int_{-\infty}^{\infty} x(t) x(-(\tau-t)) dt$

$y(t) = x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

$y(\tau) = x(\tau) \otimes h(\tau) = \int_{-\infty}^{\infty} x(t) h(\tau-t) dt$

$\Rightarrow R_x(\tau) = x(\tau) \otimes x(-\tau)$

$\mathcal{F}\{R_x(\tau)\} = X(f) X(-f)$

$x(t)$  reel  $\Rightarrow X(f) = X^*(-f) = X(f) X^*(f) = |X(f)|^2 = E_x(f)$

$\Rightarrow X(-f) = X^*(f)$

$R_x(\tau) \xleftrightarrow{\mathcal{F}} E_x(f)$



$$R_x(\tau) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt$$

$$R_x(0) = \int_{-\infty}^{\infty} |x(t)|^2 dt = E_x = \int_{-\infty}^{\infty} \underbrace{|X(f)|^2}_{E_x(f)} df = E_x$$

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) \otimes h(t)$$

$$Y(f) = X(f) \cdot H(f) \Rightarrow \underbrace{|Y(f)|^2}_{E_y(f)} = \underbrace{|X(f)|^2}_{E_x(f)} \cdot |H(f)|^2$$

$$E_y(f) = E_x(f) \cdot |H(f)|^2$$

$$\uparrow \mathcal{F}^{-1}$$

$$R_y(\tau) = R_x(\tau) \otimes R_h(\tau)$$

$$R_y(\tau) = y(\tau) \otimes y^*(-\tau) = R_x(\tau) \otimes R_h(\tau)$$

Örnek:

$$x(t) = e^{-at} u(t)$$

a)  $E_x(f) = ?$ ,  $E_x = ?$

$$E_x(f) = |X(f)|^2 \Rightarrow X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j2\pi ft} dt$$

$$= \int_0^{\infty} e^{-(a+j2\pi f)t} dt = \frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \Big|_0^{\infty} = \frac{e^{-\infty} - e^{-0}}{-(a+j2\pi f)} = \frac{1}{a+j2\pi f}$$

$$X(f) = \frac{1}{a+j2\pi f} \rightarrow E_x(f) = |X(f)|^2 = \frac{1}{(a)^2 + (2\pi f)^2}$$

$$E_x = \int_{-\infty}^{\infty} E_x(f) df = \int_{-\infty}^{\infty} \frac{df}{a^2 + (2\pi f)^2} = \frac{1}{2\pi a} \arctan\left(\frac{2\pi f}{a}\right) \Big|_{-\infty}^{\infty} = \frac{1}{2\pi a} \left[ \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) \right] = \frac{1}{2a}$$

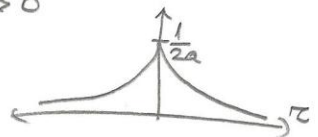
$$\int \frac{dx}{a^2 + bx^2} = \frac{1}{ab} \arctan\left(\frac{bx}{a}\right)$$

b)  $R_x(\tau) = ?$ ;  $E_x(f) = ?$ ;  $E_x = ?$

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-a(t-\tau)} u(t-\tau) dt = \int_{\tau}^{\infty} e^{-at} e^{-a(t-\tau)} dt, \tau > 0$$

$$= e^{a\tau} \int_{\tau}^{\infty} e^{-2at} dt = e^{a\tau} \frac{e^{-2at}}{-2a} \Big|_{\tau}^{\infty} = e^{a\tau} \frac{e^{-\infty} - e^{-2a\tau}}{-2a} = \frac{e^{-a\tau}}{2a}, \tau > 0$$

$$R_x(\tau) = \frac{e^{-2a|\tau|}}{2a}, \text{ çift fonk } \forall \tau$$



$$E_x = R_x(\tau) \Big|_{\tau=0} = R_x(0) = \frac{1}{2a}$$

$$E_x(f) = \mathcal{F}\{R_x(\tau)\} = \frac{1}{a^2 + (2\pi f)^2}$$

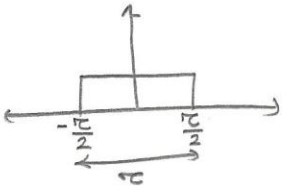
$$\text{Tablodan: } e^{-a|\tau|}, a > 0 \xleftrightarrow{\mathcal{F}} \frac{2a}{a^2 + (2\pi f)^2}$$

$$BG = \infty \text{ (örneğin } \text{sinc}(f))$$

• Asıl BG (Essential BW): İşaretin enerjisinin %90,95,99'unu içeren BG olarak tanımlanır.

Örnek:

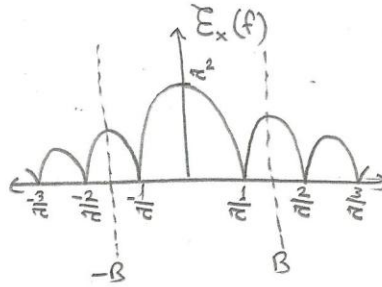
$x(t) = \text{II}\left(\frac{t}{\tau}\right)$ 'nin %90 bant genişliğini bulunuz.



$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\tau/2}^{\tau/2} 1^2 dt = \tau \text{ (%100)}$$

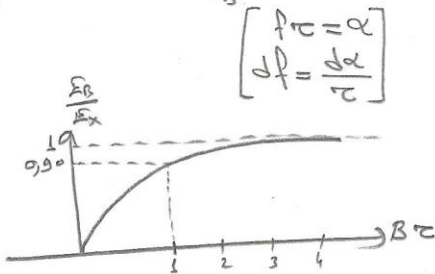
$$X(f) = \tau \text{sinc}(f\tau)$$

$$E_x(f) = |X(f)|^2 = \tau^2 \text{sinc}^2(f\tau)$$



$$E_x = \int_{-\infty}^{\infty} E_x(f) df \Rightarrow \%100 \rightarrow \tau$$

$$E_B = \int_{-B}^B E_x(f) df = \tau^2 \int_{-B}^B \text{sinc}^2(f\tau) df = \tau \int_{-B\tau}^{B\tau} \text{sinc}^2(\alpha) d\alpha \Rightarrow \frac{E_B}{\tau} = \frac{E_B}{E_x} = \int_{-B\tau}^{B\tau} \text{sinc}^2(\alpha) d\alpha$$



$$B\tau = 1 \Rightarrow \frac{E_B}{E_x} = 0,9098$$

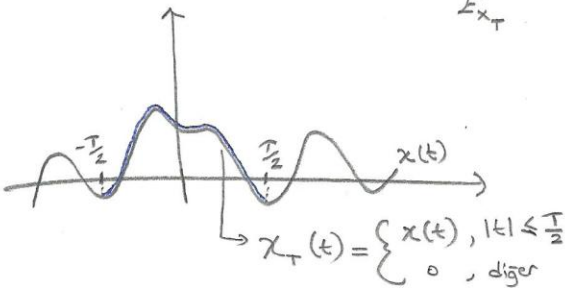
$$E_B = 0,90 E_x$$

$$B\tau = 1 \Rightarrow B = \frac{1}{\tau} \text{ işaretin enerjisinin \%90'ünü içeren Bant Genişliği.}$$

• Güç ve Güç Yoğunluk Fonksiyonu (GYF)

(Power Spectral Density - PSD)

$$x(t) \text{ real} \Rightarrow P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt = \lim_{T \rightarrow \infty} \frac{E_{x_T}}{T}$$



$$x_T(t) \xrightarrow{\mathcal{F}} X_T(f)$$

$$E_{x_T} = \int_{-T/2}^{T/2} x^2(t) dt = \int_{-T/2}^{T/2} x_T^2(t) dt = \int_{-\infty}^{\infty} |X_T(f)|^2 df$$

$$P_x = \lim_{T \rightarrow \infty} \frac{E_{x_T}}{T} = \lim_{T \rightarrow \infty} \frac{1}{T} \left[ \int_{-\infty}^{\infty} |X_T(f)|^2 df \right]$$

$$T \uparrow \Rightarrow x_T(t) \text{'nin süresi de} \uparrow \Rightarrow E_{x_T} \text{ de} \uparrow \Rightarrow |X_T(f)|^2 \text{ de} \uparrow$$

$$= \int_{-\infty}^{\infty} \left[ \lim_{T \rightarrow \infty} \frac{1}{T} |X_T(f)|^2 \right] df$$

Güç Yoğunluğu Fonksiyonu - GYF  $S_x(f)$

$$P_x = \int_{-\infty}^{\infty} S_x(f) df, \quad S_x = \lim_{T \rightarrow \infty} \frac{1}{T} |X(f)|^2$$

• Güç İfadelerinin Özetliki Fonksiyonu

$$x(t) \text{ güç işareti} \Rightarrow R_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x^*(t-\tau) dt$$

$$R_x(\tau) = R_x(-\tau)$$

$$x(t) \text{ reel} \Rightarrow R_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x(t-\tau) dt$$

$$\text{Enerji} \Rightarrow \int_{-\infty}^{\infty} x_T(t) x_T(t-\tau) dt = R_{x_T}^e(\tau)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} R_{x_T}^e(\tau)$$

$$R_{x_T}^e(\tau) \xrightarrow{F} |X_T(f)|^2$$

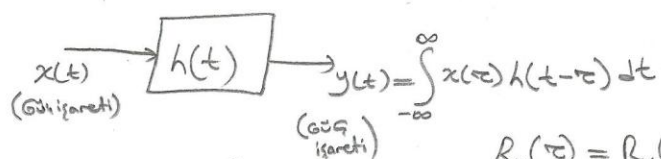
$$F \{ R_x(\tau) \} = \lim_{T \rightarrow \infty} \frac{|X_T(f)|^2}{T} = S_x(f)$$

$$R_x(\tau) \xrightarrow{F} S_x(f)$$

$$R_x(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = P_x$$

$$R_x(\tau) \xrightarrow{F} S_x(f) \Rightarrow R_x(\tau) \Big|_{\tau=0} = \int S_x(f) e^{j2\pi f \tau} df \Big|_{\tau=0}$$

$$\Rightarrow R_x(0) = \int_{-\infty}^{\infty} S_x(f) df = P_x$$



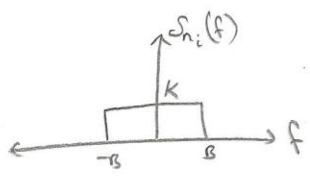
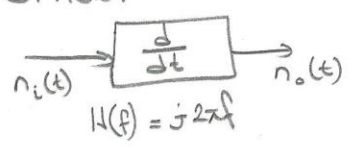
$$R_y(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} y(t) y^*(t-\tau) dt$$

$$R_y(\tau) = R_x(\tau) \otimes h(\tau) \otimes h^*(-\tau)$$

$$S_y(f) = S_x(f) \cdot H(f) \cdot H^*(f) = S_x(f) |H(f)|^2$$

$$S_y(f) = S_x(f) |H(f)|^2$$

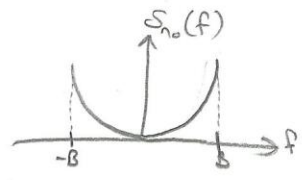
— Örnek:



$n_i(t)$ , GYF  $S_{n_i}(f) = \begin{cases} K, & |f| \leq B \\ 0, & \text{diğer} \end{cases}$  olan bir görülmüş işarettir.

$S_{n_o}(f) = ?$ , Çıktadaki güç  $P_{n_o} = ?$

$$S_{n_o}(f) = S_{n_i}(f) \cdot |H(f)|^2 = \begin{cases} K(2\pi f)^2, & |f| \leq B \\ 0, & \text{diğer} \end{cases}$$



$$P_{n_o} = \int_{-\infty}^{\infty} S_{n_o}(f) df = \int_{-B}^B 4\pi^2 f^2 K df = \frac{8K\pi^2 B^3}{3}$$

Örnek:

$x(t)$  periyodik ise  $R_x(\tau) = ?$   $S_x(f) = ?$   
 güç işaretidir!

$$R_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) x^*(t-\tau) dt$$

$x(t)$  periyodik ise  $\Rightarrow x(t) = \sum_n x_n e^{j2\pi n f_0 t}$

$$= \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \underbrace{\sum_n x_n e^{j2\pi n f_0 t}}_{x(t)} \underbrace{\sum_m x_m^* e^{-j2\pi m f_0 (t-\tau)}}_{x^*(t-\tau)} dt$$

$$= \frac{1}{T_0} \sum_n \sum_m x_n x_m^* e^{j2\pi m f_0 \tau} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{j2\pi n f_0 t} e^{-j2\pi m f_0 t} dt$$

$$= \begin{cases} T_0 & n=m \\ 0 & n \neq m \end{cases}$$

$$= \frac{1}{T_0} \sum_n |x_n|^2 T_0 = \sum_n |x_n|^2 e^{j2\pi n f_0 \tau}, \quad n=m$$

$$R_x(\tau) = \sum_n |x_n|^2 e^{j2\pi n f_0 \tau} \xrightarrow{\mathcal{F}} S_x(f) = \sum_n |x_n|^2 \delta(f - n f_0)$$

02.04.2012

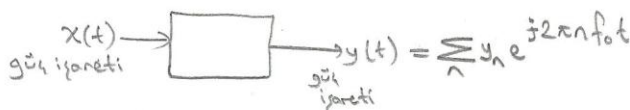
$x(t)$  periyodik

$$\Rightarrow R_x(\tau) = \frac{1}{T_0} \int_{T_0} x(t) x^*(t-\tau) dt = \sum_n |x_n|^2 e^{j2\pi n f_0 \tau} \xrightarrow{\mathcal{F}} S_x(f) = \sum_n |x_n|^2 \delta(f - n f_0)$$

$$\rightarrow P_x = \int_{-\infty}^{\infty} S_x(f) df = \int_{-\infty}^{\infty} \sum_n |x_n|^2 \delta(f - n f_0) df = \sum_n |x_n|^2 \underbrace{\int_{-\infty}^{\infty} \delta(f - n f_0) df}_1 = \sum_n |x_n|^2$$

$x(t)$  güç işareti

$$\Rightarrow P_x = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \int_{-\infty}^{\infty} S_x(f) df = \sum_n |x_n|^2$$



$$S_x(f) = \sum_n |x_n|^2 \delta(f - n f_0) \quad S_y(f) = S_x(f) \cdot |H(f)|^2$$

$$= |H(f)|^2 \sum_n |x_n|^2 \delta(f - n f_0)$$

$$= \sum_n |x_n|^2 \underbrace{|H(f)|^2}_{\downarrow} \delta(f - n f_0)$$

$$= \sum_n \underbrace{|x_n|^2 |H(n f_0)|^2}_{|y_n|^2} \delta(f - n f_0)$$

$$P_y = \int_{-\infty}^{\infty} S_y(f) df = \sum_n |y_n|^2 = \sum_n |x_n|^2 |H(n f_0)|^2$$



Örnek: Güç yoğunluk fonksiyonlarını bulunuz.

a)  $x(t) = K e^{j2\pi f_0 t} \rightarrow S_x(f) = |K|^2 \delta(f-f_0)$

b)  $y(t) = A \cos(2\pi f_0 t) = \frac{A}{2} [e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}] \rightarrow S_y(f) = \frac{|A|^2}{2^2} [\delta(f-f_0) + \delta(f+f_0)]$

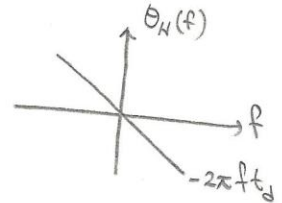
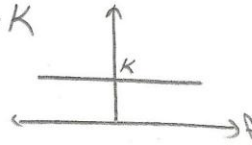
c)  $z(t) = A \sin(2\pi f_0 t) \rightarrow S_z(f) = S_y(f)$

Bozulmasız İletim: (Distortionless transmission)

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = K x(t-t_d)$  ;  $k=sbt$ ,  $t_d=sbt$   
kanal

$Y(f) = K \cdot X(f) e^{-j2\pi f t_d}$

$H(f) = \frac{Y(f)}{X(f)} = K e^{-j2\pi f t_d} \rightarrow |H(f)| = K$

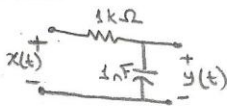


İdeal bir kanalın genlik spektrumu sabit, faz spektrumu lineer olmalı.

$t_d(f) = -\frac{1}{2\pi} \frac{d\theta_H(f)}{df}$

↳ Grup gecikmesi (Group delay, Envelope delay)

Örnek:



$H(f) = ?$   $\theta_H(f) = ?$   $t_d(f) = ?$

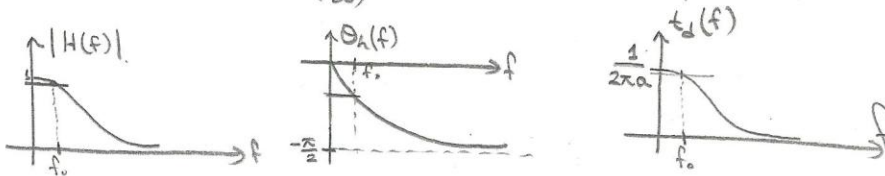
Bu filtreden işaretin bozulmadan geçmesi için  $x(t)$ 'in BG ne olmalı?

(%2'lik genlik cevabı toleransı, %5'lik grup gecikmesi toleransı mevcuttur)

$H(f) = \frac{Y(f)}{X(f)} = \frac{1}{R + \frac{1}{j2\pi f C}} = \frac{1}{1 + j2\pi f RC} = \frac{1}{1 + j\frac{f}{a}}$  ;  $a = \frac{1}{2\pi RC} = \frac{10^6}{2\pi}$   
 $= \frac{a}{a + jf}$

$|H(f)| = \frac{a}{\sqrt{a^2 + f^2}}$  ,  $t_d(f) = -\frac{1}{2\pi} \frac{d\theta_H(f)}{df} = \frac{1}{2\pi} \frac{a}{a^2 + f^2}$

$\theta_H(f) = 0 - \arctan\left(\frac{f}{a}\right)$



BG{X(f)} = f\_0

$H(0) = 1$  ,  $|H(f_0)| = 0,98 \Rightarrow \frac{a}{a^2 + f_0^2} \geq 0,98$  (1)

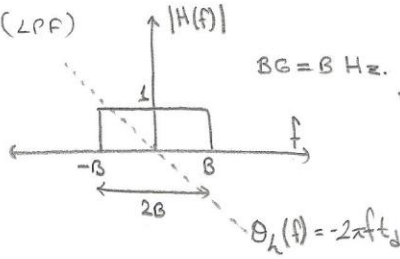
$t_d(0) = \frac{1}{2\pi a}$  ,  $t_d(f_0) \geq \frac{0,95}{2\pi a} \Rightarrow \frac{1}{2\pi} \frac{a}{a^2 + f_0^2} \geq \frac{0,95}{2\pi a}$  (2)

(1)  $\Rightarrow f_0 \leq 32,32 \text{ kHz}$   
(2)  $\Rightarrow f_0 \leq 36,5 \text{ kHz}$  } BG{X(t)} = 32,32 \text{ kHz (her ikisi için de uygun f\_0)}

$y(t) = 1 x(t-t_d)$  ;  $t_d = \frac{1}{2\pi a} \approx 10^{-6} \Rightarrow y(t) \approx x(t-10^{-6})$

• İdeal ve Pratik Filtreler:

AGF (LPP)



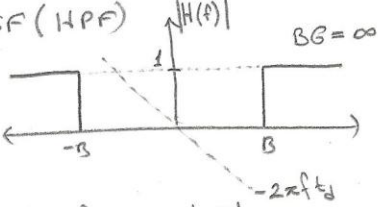
BG = B Hz.

$$\Rightarrow H(f) = |H(f)| e^{j\theta_h(f)}$$

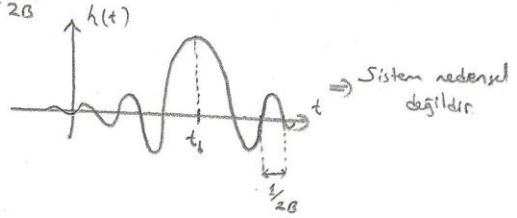
$$H(f) = \text{rect}\left(\frac{f}{2B}\right) e^{-j2\pi f t_d}$$

$$h(t) = 2B \text{sinc}[2B(t-t_d)]$$

YGF (HPF)

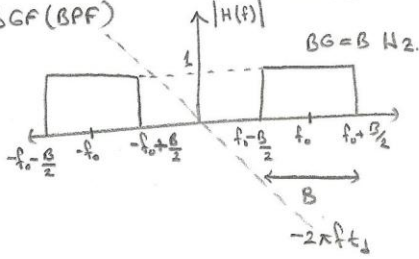


$2Bt = k ; k \in \mathbb{Z} - \{0\}$   
 $t = \frac{k}{2B}$



⇒ Sistem nedensel değildir

BGF (BPF)



Nedensellik Şartı → t < 0'da h(t) = 0 olmalı.

Paley-Wiener Kriteri t < 0, h(t) = 0.

$$\int_{-\infty}^{\infty} \frac{|\ln|H(f)||}{1+f^2} df < \infty$$

1)  $\hat{h}(t) = h(t) \cdot u(t)$

↳ Nedenseldir. → pencere fonksiyonu.

→  $\hat{h}(t) = h(t) \cdot w(t)$

2)  $t_d \gg$  olduğunda nedensel kabul edilebilir.

$t_d \approx \frac{3}{4} \frac{1}{2B} \Rightarrow h(t)$  'yi ideale yakın gerçekleştirir.

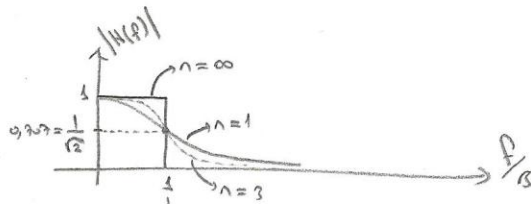
Ses işareti için BG = 20 kHz ise

$t_d = 3 \frac{1}{2 \cdot 20 \cdot 10^3} = 7,5 \times 10^{-5} \approx 10^{-4} = 0.1 \text{ms}$

• Pratik Filtreler:

Butterworth

$|H(f)| = \frac{1}{\sqrt{1 + (\frac{f}{B})^{2n}}}$



↳ Kesim frekansı: Güç yarıya; genlik de 1/sqrt(2)'sine düşer.

Half-power BW = 3dB BW

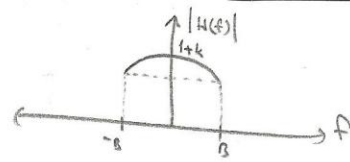
• Bir Haberleşme Kanalında İşaret Bozulması,

→ Lineer Bozulma (Distortion)

Kanal ideal değilse lineer bozulma olur (yayılmaz)

Örnek:

$$H(f) = \begin{cases} (1+k\cos(2\pi fT)) e^{-j2\pi f t_d}, & |f| < B \\ 0, & |f| > B \end{cases}$$



BG = B Hz olan  $x(t)$  işareti bu filtrenin girişine uygulanmaktadır.  $y(t)$  çıkışını bulunuz.

$x(t) \rightarrow h(t) \rightarrow y(t) = x(t) \otimes h(t)$

$Y(f) = X(f) \cdot H(f)$

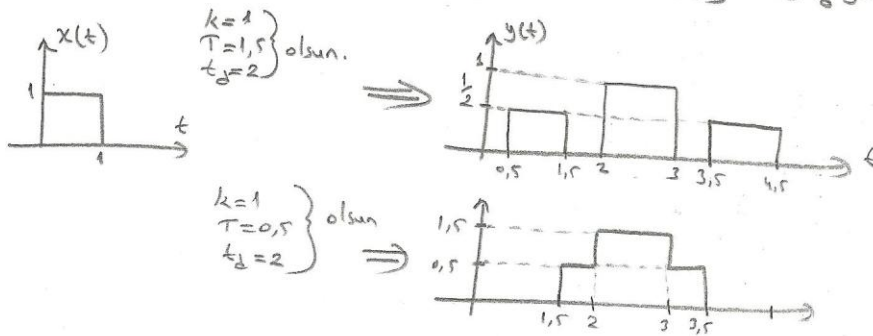
$= X(f) \cdot [1+k\cos(2\pi fT)] e^{-j2\pi f t_d}$

$= X(f) e^{-j2\pi f t_d} + k X(f) \frac{1}{2} [e^{j2\pi f T} + e^{-j2\pi f T}] e^{-j2\pi f t_d}$

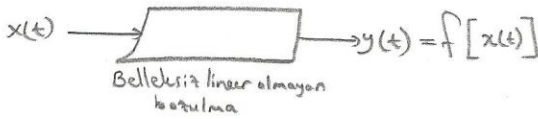
$\downarrow \mathcal{F}^{-1}$

$y(t) = x(t-t_d) + \frac{k}{2} [x(t-T-t_d) + x(t+T-t_d)]$

$x(t) + \frac{k}{2} [x(t-T) + x(t+T)] \rightarrow t_d$  gecikmesi.



→ Lineer Olmayan Bozulma:



Maclaurin serisine açılırsa:

$y(t) = f[x(t)] = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \dots + a_k x^k(t) + \dots$

BG  $\{x(t)\} = B$  Hz. BG  $\{x^k(t)\} = k \cdot B$  Hz. → frekansta yayılma (spectral dispersion)

Örnek:

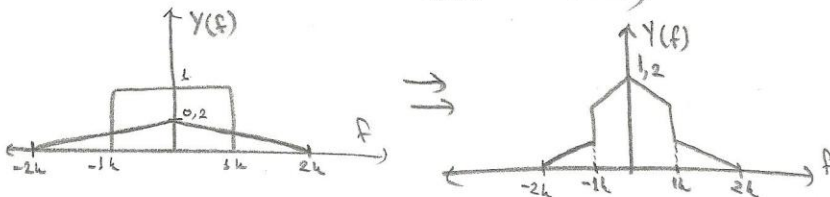
$x(t) \rightarrow y(t) = x(t) + 0.0001 x^2(t)$ ,  $x(t) = 2000 \text{sinc}(2000t)$

$y(t) = ?$ ;  $Y(f) = ?$ ; BG  $\{y(t)\} = ?$ ;  $x(t)$  işareti  $y(t)$  den bozulması olarak elde edilebilir mi?

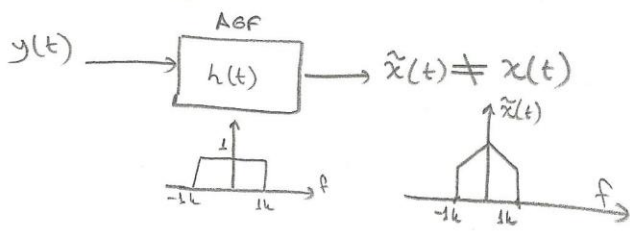


$y(t) = 2000 \text{sinc}(2000t) + 400 \text{sinc}^2(2000t)$

$\Rightarrow Y(f) = \text{rect}\left(\frac{f}{2000}\right) + 400 \frac{1}{2000} \Lambda\left(\frac{f}{2000}\right)$



BG  $\{y(t)\} = 2000$  Hz.



04.04.2012

## Genlik Modülasyonu (GM)

→ Modülasyon: Kanala uygun hale getirilmesi.

→ Mesaj (haber) işareti:

Temel band işareti (baseband) } → Temel band haberleşmesi  
 Ses ⇒ 0-4 kHz  
 Görüntü ⇒ 0-5/6 MHz  
 Yapı itibarıyla analog işaretlerdir. } Modülasyon → Taşıyıcı haberleşme  
 (carrier communication)  
 bandpass communication

$m(t)$ : mesaj işareti

$c(t)$ : taşıyıcı işaret

$BB\{m(t)\} = B$  Hz olsun. Yani  $M(f) = 0, |f| > B$

$$c(t) = A_c \cdot \cos(2\pi f_c t)$$

$\downarrow$  genlik (amplitude)       $\downarrow$  frekans

→ Modülasyonun Sebepleri:

- 1) Frekans bölmei çözümlenmesi (Frequency division multiplexing) yapmak için.
- 2) Verici yapısını basitleştirmek için

Antenin boyu  $= l \propto \frac{\lambda}{4}$  → dalga boyu.

Ses haberleşmesi:  $BB \approx 3$  kHz  
 $f = 3$  kHz

$$c = \lambda f \Rightarrow \lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{3 \cdot 10^3 \text{ 1/s}} = 10^5 \text{ m} = 100 \text{ km} \Rightarrow l \approx 25 \text{ km}$$

Genlik Modülasyonu (GM)

Taşıyıcı frekans  $f_c = 1$  MHz

$$\lambda = \frac{3 \cdot 10^8 \text{ m/s}}{10^6 \text{ 1/s}} = 300 \text{ m} \Rightarrow l \approx 75 \text{ m}$$

- 3) Mesaj işaretinin bantgenişliğini artırarak gürültü ve karışım bağımsızlığı sağlamak.  
 (noise) (interference)

Bu açı mod ve yayılı spektrum mod. için geçerlidir.  
 (angle) (spread spectrum)

→ GM (AM) - linear mod.

→ Açı (angle) - Nonlinear/exponential mod

↳ Frekans modülasyonu (FRM-FM)  
 ↳ Faz modülasyonu (FZM-PM)



Tayıyıcı Bastırılmış Gıft Yan Bantlı Genlik Modölasyonu (TB-CYB-GM)

(Double sideband suppressed carrier AM) - (DSB-SC-AM)

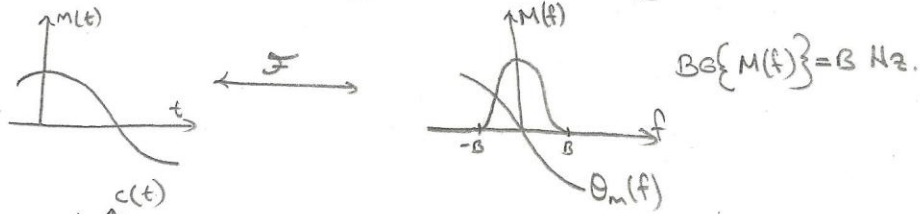
$m(t)$ : mesal (haber) işareti  $\rightarrow$  Modüle eden işaret (Modulating signal)

$c(t)$ : Tayıyıcı işaret  $\rightarrow A_c \cdot \cos(2\pi f_c t) \rightarrow$  modüle edilmiş işaret (Modulated signal)

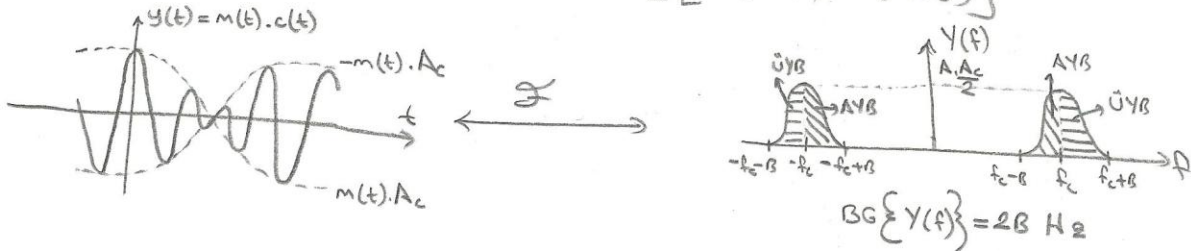
$m(t) \rightarrow \otimes \rightarrow y(t) = A_c m(t) \cos(2\pi f_c t)$

$c(t) = A_c \cos(2\pi f_c t)$

$y(t) \xrightarrow{\mathcal{F}} Y(f) = \frac{A_c}{2} [M(f-f_c) + M(f+f_c)]$



$c(t) = A_c \cos(2\pi f_c t) \xrightarrow{\mathcal{F}} C(f) = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)]$



$\rightarrow$  Demodölasyon (Faz uyumlu/Tahminli demodölasyon)  
Phase coherent demodulation

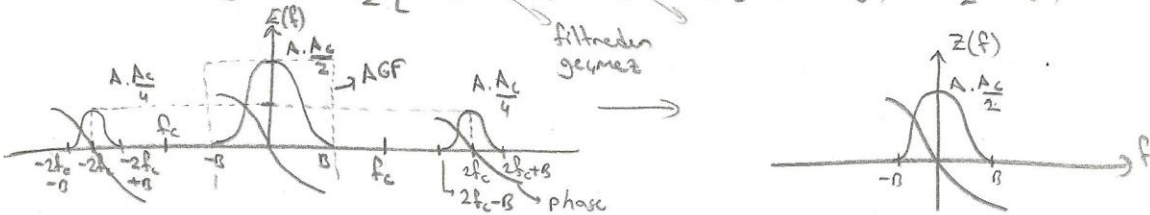
$y(t) = m(t) \cdot c(t) = A_c m(t) \cos(2\pi f_c t) \rightarrow \otimes \xrightarrow{\cos(2\pi f_c t)} \text{AGF} \rightarrow z(t) = \frac{A_c}{2} m(t)$

$\cos(2\pi f_c t)$   
 $\rightarrow$  Alıcıda üretilmiş (faz tahmini yapılmış)

$e(t) = y(t) \cos(2\pi f_c t) = A_c m(t) \cos^2(2\pi f_c t)$   
 $\bullet \cos^2(x) = \frac{1}{2} [1 + \cos(2x)]$

$e(t) = \frac{A_c}{2} m(t) + \frac{A_c}{2} m(t) \cos(2\pi 2f_c t)$

$E(f) = \frac{A_c}{2} M(f) + \frac{A_c}{2} [M(f-2f_c) + M(f+2f_c)] \Rightarrow Z(f) = \frac{A_c}{2} M(f) \xrightarrow{\mathcal{F}} z(t) = \frac{A_c}{2} m(t)$



$2f_c - B \geq B$  olmalı.

$2f_c \geq 2B$

$f_c \geq B \rightarrow$  bu şart sağlanmazsa işaret düzgün elde edilemez.

→ Faz Tahmini

- 1) Pilot taşıyıcı kullanılmak
- 2) Phase locked loop (PLL) devresi kullanılmak.

Darbantlı işaret  $f_c \gg B$

$$\frac{f_c}{B} \gg 1$$

Radio yayını (GM) :  $f_c = 550 \text{ kHz} - 1600 \text{ kHz}$

$$100 < \frac{f_c}{B} < 300$$

03.04.2012

TB-G4B-GM

$$y(t) = m(t) \cdot c(t) = A_c m(t) \cos(2\pi f_c t)$$

$$e(t) = y(t) \cos(2\pi f_c t) \rightarrow \text{AGF} \rightarrow z(t) = \frac{A_c}{2} m(t)$$

Tahmin.

• TB-G4B-GM'nin Gücü:

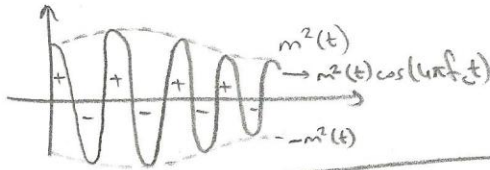
$$y(t) = m(t) \cdot c(t) = A_c m(t) \cos(2\pi f_c t)$$

$$P_y = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 m^2(t) \cos^2(2\pi f_c t) dt$$

$\xrightarrow{\frac{1}{2}[1 + \cos(2\pi 2f_c t)]}$

$$= \frac{1}{2} A_c^2 \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} [m^2(t) + m^2(t) \cos(4\pi f_c t)] dt$$

$\approx 0$ , yüksek frekans  
cos, m(t) ye göre çok hızlı değişiyor



\* + & - : bölgelerin alanları

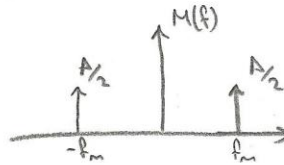
$$P_y = \frac{A_c^2}{2} P_m$$

$$P_m = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) dt$$

Örnek: Ton Modülasyonu.

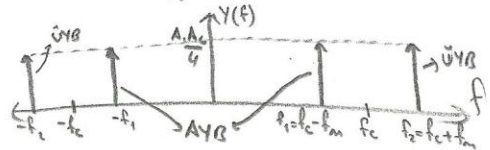
$$m(t) = A \cos(2\pi f_m t), f_m \ll f_c$$

$$M(f) = \frac{A}{2} [M(f-f_m) + M(f+f_m)]$$



$$y(t) = m(t) \cdot c(t) = A \cdot A_c \cdot \cos(2\pi f_m t) \cos(2\pi f_c t) = A \cdot \frac{A_c}{2} [\cos(2\pi(f_c - f_m)t) + \cos(2\pi(f_c + f_m)t)]$$

$$Y(f) = \frac{A \cdot A_c}{4} [\delta(f-f_1) + \delta(f+f_1) + \delta(f-f_2) + \delta(f+f_2)]$$



$$y_{A4B}(t) = \frac{A \cdot A_c}{2} \cos(2\pi(f_c - f_m)t) \rightarrow P_{A4B} = \frac{A^2 A_c^2}{4} \cdot \frac{1}{2}$$

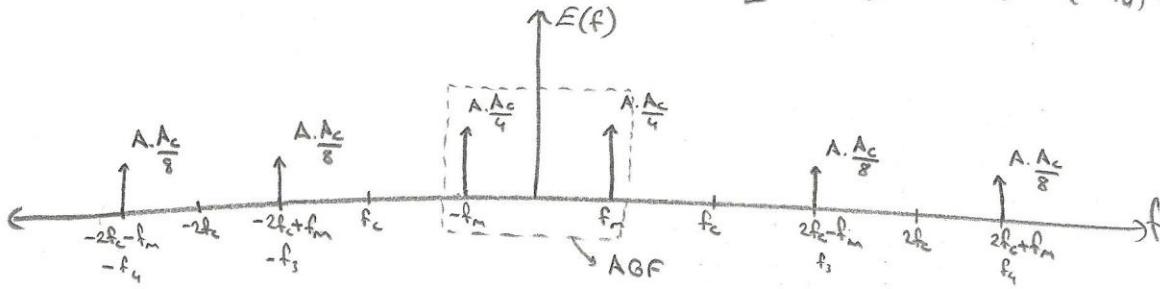
$$y_{Ü4B}(t) = \frac{A \cdot A_c}{2} \cos(2\pi(f_c + f_m)t) \rightarrow P_{Ü4B} = \frac{A^2 A_c^2}{4} \cdot \frac{1}{2}$$

→ Demodülasyon Adımı:

$$\begin{aligned}
 e(t) &= y(t) \cos(2\pi f_c t) = \frac{A \cdot A_c}{2} [\cos(2\pi f_1 t) + \cos(2\pi f_2 t)] \cos(2\pi f_c t) \\
 &= \frac{A \cdot A_c}{4} [\underbrace{\cos(2\pi(f_c - f_1)t)}_{\cos 2\pi f_m t} + \underbrace{\cos(2\pi(f_c + f_1)t)}_{2f_c - f_m = f_3} + \underbrace{\cos(2\pi(f_c - f_2)t)}_{\cos 2\pi f_m t} + \underbrace{\cos(2\pi(f_c + f_2)t)}_{2f_c + f_m = f_4}] \\
 &= \frac{A \cdot A_c}{4} [2\cos(2\pi f_m t) + \cancel{\cos(2\pi f_3 t)} + \cancel{\cos(2\pi f_4 t)}]
 \end{aligned}$$

AGF'ler gider.

$$E(f) = \frac{A \cdot A_c}{4} [\delta(f - f_m) + \delta(f + f_m)] + \frac{A \cdot A_c}{8} [\delta(f - f_3) + \delta(f + f_3) + \delta(f - f_4) + \delta(f + f_4)]$$



$$e(t) \xrightarrow{\text{AGF } BGF=f_m} z(t) = \frac{A \cdot A_c}{2} \cos(2\pi f_m t)$$

$$z(f) = \frac{A \cdot A_c}{4} [\delta(f - f_m) + \delta(f + f_m)]$$

$$P_s = \left(\frac{A \cdot A_c}{2}\right)^2 \left[\frac{1}{2} + \frac{1}{2}\right] = \frac{A^2 A_c^2}{4} = \frac{A^2}{2} P_m ; P_m = \frac{A^2}{2}$$

Örnek:

$$m(t) = \text{sinc}(10^4 t)$$

TB-CYB-GM işaretini bulunuz. BG'ni bulunuz. Taşıyıcı işaretin frekansı = 1 MHz

$$c(t) = A_c \cos(2\pi 10^6 t)$$

$$m(t) = \text{sinc}(10^4 t)$$

$$y(t) = m(t) \cdot c(t) = A_c \cos(2\pi 10^6 t) \text{sinc}(10^4 t)$$

$$BG\{y(t)\} = 2B = 2BG\{m(t)\}$$

$$A \cos(t) \xrightarrow{\mathcal{F}} A \pi \left(\frac{f}{c}\right)$$

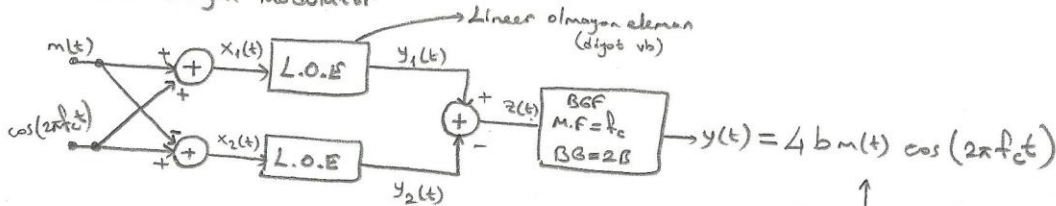
$$\text{sinc}\left(\frac{10^4 t}{c}\right) \xrightarrow{\mathcal{F}} \frac{1}{10^4} \pi \left(\frac{f}{10^4}\right) \rightarrow B = \frac{10^4}{2} \text{ Hz}$$

$$BG\{y(t)\} = 2B = 10^4 \text{ Hz} = 10 \text{ kHz}$$

## Modülatörler

→ Çarpıcı modülatör (multiplier modulator)

→ Lineer olmayan modülatör



$$x_1(t) = m(t) + \cos(2\pi f_c t)$$

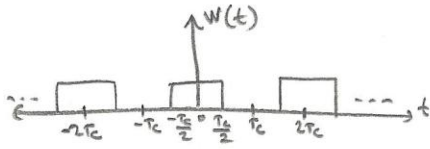
$$x_2(t) = -m(t) + \cos(2\pi f_c t)$$

$$y(t) = a x(t) + b x^2(t)$$

$$z(t) = y_1(t) - y_2(t) = 2a m(t) + 4b m(t) \cos(2\pi f_c t)$$

BGF geçirmedi.

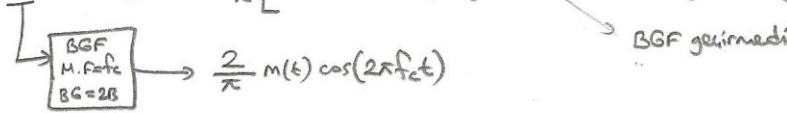
→ Anahtarlama Modülasyonu (switching mod.)



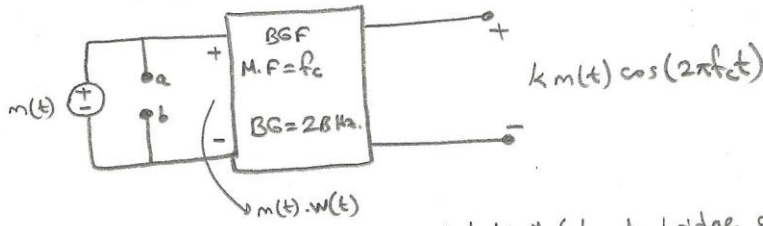
FS aılımı  

$$w(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(2\pi f_c t) - \frac{1}{3} \cos(2\pi 3f_c t) + \frac{1}{5} \cos(2\pi 5f_c t) - \dots \right]$$

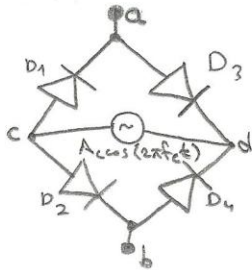
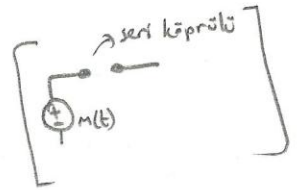
$$m(t) \cdot w(t) = \frac{1}{2} m(t) + \frac{2}{\pi} \left[ m(t) \cos(2\pi f_c t) - \frac{1}{3} m(t) \cos(2\pi 3f_c t) + \frac{1}{5} m(t) \cos(2\pi 5f_c t) - \dots \right]$$



$BG \{m(t)\} = B \text{ Hz.}$



⇒ Paralel köprülü diyot modülasyonu (shunt bridge diode modülasyonu)



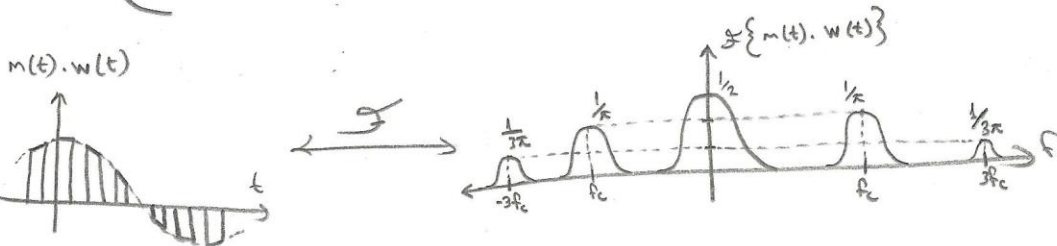
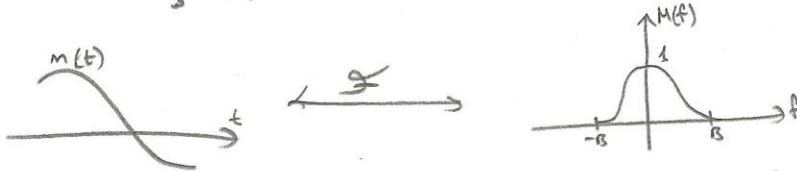
$D_1 \equiv D_2$   
 $D_3 \equiv D_4$

$V_c > V_d \Rightarrow$  bütün diyotlar iletimde (kısa devre olur.)

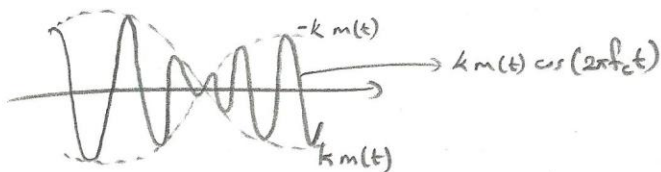
$V_c < V_d \Rightarrow$  tüm diyotlar kesimde (açık devre olur.)

$T_c$  ile periyodik olarak bu diyotlar bir açılıp bir kapanacaktır.

↳ Taşıyıcı olmadığı için tek dengeli modülasyondur.

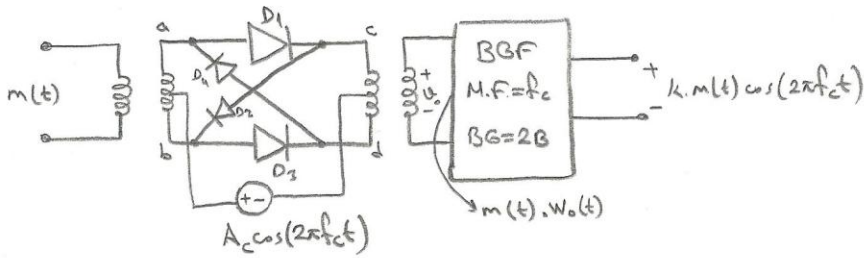


$k \cdot m(t) \cos(2\pi f_c t)$



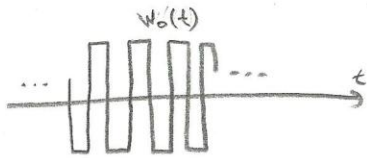


→ Gift Dengeli Modölatör (Ring Mod.)



Taşıyıcının pozitif yarı periyodunda;  $D_1, D_3$  iletimde  $\Rightarrow a=c \rightarrow \varphi_i(t), m(t)$  ile orantılı.  
 $D_2, D_4$  kesimde  $b=d$

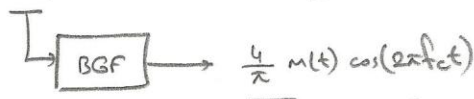
Taşıyıcının negatif yarı periyodunda;  $D_1, D_3$  kesimde  $\Rightarrow a=d \rightarrow \varphi_i(t), -m(t)$  ile orantılı.  
 $D_2, D_4$  iletimde  $b=c$



FS açılımı

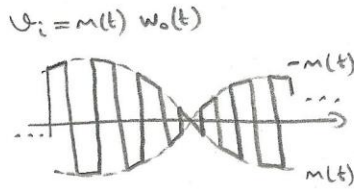
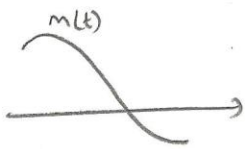
$$w_o(t) = \frac{4}{\pi} \left[ \cos(2\pi f_c t) - \frac{1}{3} \cos(2\pi 3f_c t) + \frac{1}{5} \cos(2\pi 5f_c t) - \dots \right]$$

$$\varphi_i(t) = m(t) \cdot w_o(t) = \frac{4}{\pi} \left[ m(t) \cos(2\pi f_c t) - \frac{1}{3} m(t) \cos(2\pi 3f_c t) + \frac{1}{5} m(t) \cos(2\pi 5f_c t) - \dots \right]$$



BGF geçirmes.

↳ ne "m(t)" ne de "cos(2πfct)" olmadısı için Gift dengeli modölatör denir.

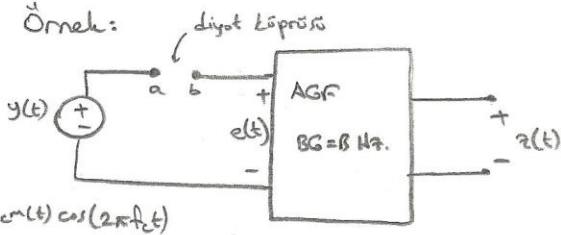


• Demodölatör :

Çarpıcı + AGF

modölatör yapısında kullanılan çarpıcılar burada da kullanılabilir.

Örnek:



$$e(t) = y(t) w(t)$$

$$= A_c m(t) \cos(2\pi f_c t) \left[ \frac{1}{2} + \frac{2}{\pi} \left( \cos 2\pi f_c t - \frac{1}{3} \cos 2\pi 3f_c t + \dots \right) \right]$$

$$= \frac{A_c}{\pi} 2 m(t) \cos^2(2\pi f_c t) + k \cdot m(t) \cos(2\pi n f_c t)$$

$\frac{1}{2} [1 + \cos(2\pi 2f_c t)]$  AGF geçirmes.

$$= \frac{2A_c}{\pi} \frac{1}{2} m(t) + \frac{A_c}{\pi} m(t) \cos(2\pi 2f_c t)$$

AGF geçirmes.

$$z(t) = \frac{A_c}{\pi} m(t)$$

● BÜYÜK TAŞIYICILI GYB-GM (BT-GYB-GM)  
[DSB-Large Carrier (LC-AM)]

Ticari GM:  
(Commercial AM)

$$y(t) = \underbrace{A_{c1} m(t) \cos(2\pi f_c t)}_{\text{TB-GYB}} + \underbrace{A_{c2} \cos(2\pi f_c t)}_{\text{BT}}$$

$$= A_{c2} \cos(2\pi f_c t) \left[ 1 + \frac{A_{c1}}{A_{c2}} m(t) \right]$$

$$y(t) = A_{c2} \left[ 1 + \frac{A_{c1}}{A_{c2}} m(t) \right] \cos(2\pi f_c t)$$

$$\Rightarrow A_{c2} \equiv A_c, \quad \frac{A_{c1}}{A_{c2}} = \frac{\mu}{m_p}$$

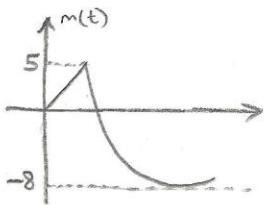
- $0 < \mu < 1$  : Modülasyon İndeksi
- $m_p = \max|m(t)|$

$$\Rightarrow m_n(t) = \frac{m(t)}{m_p} \text{ (normalleştirilmiş mesaj işareti)}$$

$$\Rightarrow y(t) = A_c \underbrace{\left[ 1 + \mu m_n(t) \right]}_{\text{modüle eden işaret}} \cos(2\pi f_c t)$$

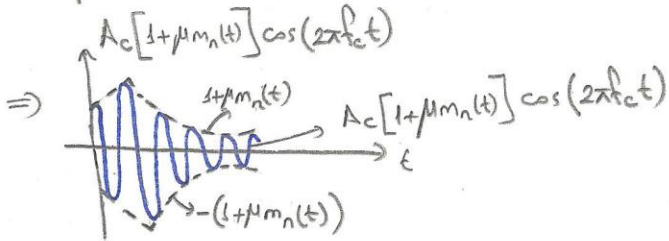
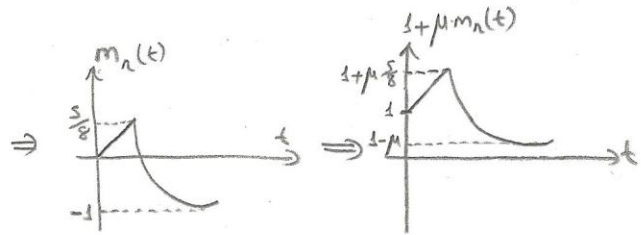
$$m_p = \max|m(t)| \Rightarrow m_n(t) = \frac{m(t)}{m_p} \rightarrow \forall t \text{ için } 1 + \mu m_n(t) \geq 0$$

— Örnek:



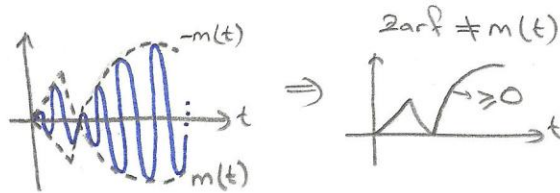
$$m_p = \max|m(t)| = 8$$

$$m_n(t) = \frac{m(t)}{m_p}$$

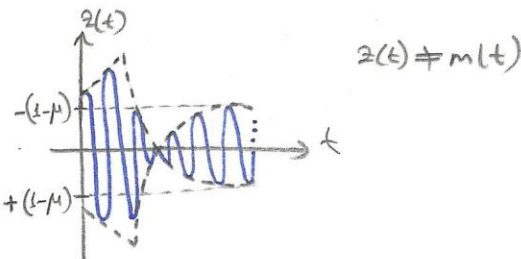


TB-GYB-GM olsaydı:

$$y(t) = A_c m(t) \cos(2\pi f_c t) \Rightarrow$$

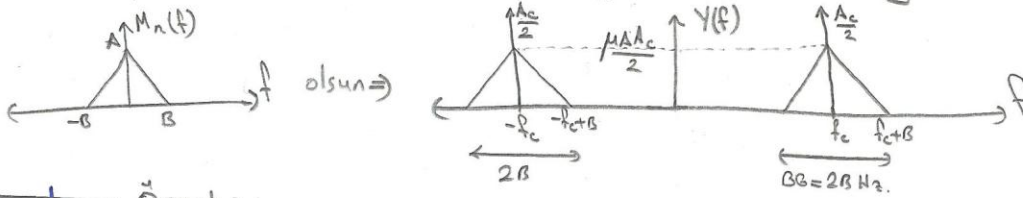


$\mu > 1$  olursa aşırı modülasyon (over modulation) denir.



$$y(t) = A_c [1 + \mu m_n(t)] \cos(2\pi f_c t) = A_c \cos(2\pi f_c t) + \mu A_c m_n(t) \cos(2\pi f_c t)$$

$$Y(f) = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)] + \mu \frac{A_c}{2} [M_n(f-f_c) + M_n(f+f_c)]$$



Örnek:

-Ton Modülasyonu-

$$m(t) = C \cdot \cos(2\pi f_m t), \quad (f_m \ll f_c)$$

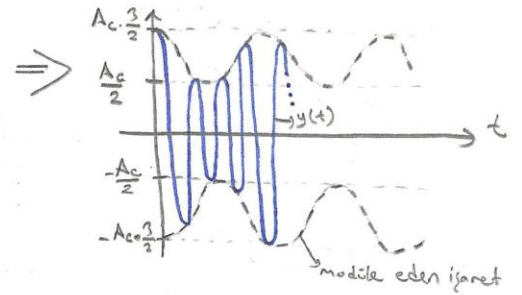
$\mu = \frac{1}{2}$  ise BT-GYB-GM işareti  $y(t)$ 'yi uzaştır.  $Y(f)$ 'i çiziniz.

Görün:  $m_p = \max|m(t)| = C$ ,  $m_n(t) = \frac{m(t)}{m_p} = \cos(2\pi f_m t)$

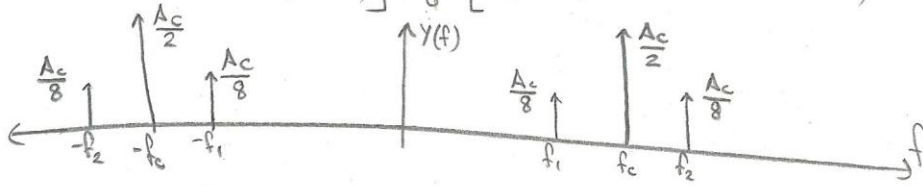
$$\Rightarrow y(t) = A_c [1 + \mu m_n(t)] \cos(2\pi f_c t) = A_c [1 + \frac{1}{2} \cos(2\pi f_m t)] \cos(2\pi f_c t) \quad \bullet f_m \ll f_c \Rightarrow T_m \gg T_c$$

$$= A_c \cos(2\pi f_c t) + \frac{A_c}{2} \cos(2\pi f_m t) \cos(2\pi f_c t)$$

$$\frac{A_c}{4} [\cos(2\pi(f_m - f_c)t) + \cos(2\pi(f_m + f_c)t)]$$



$$\Rightarrow Y(f) = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)] + \frac{A_c}{8} [\delta(f-f_1) + \delta(f+f_1) + \delta(f-f_2) + \delta(f+f_2)]$$



BT-GYB-GM'in Gücü:

$$y(t) = A_c [1 + \mu m_n(t)] \cos(2\pi f_c t)$$

$$= \underbrace{A_c \cos(2\pi f_c t)}_{\text{Taşıyıcı}} + \underbrace{A_c \mu m_n(t) \cos(2\pi f_c t)}_{\text{Yan bandlar}}$$

$$P_{\text{Taşıyıcı}} = \frac{A_c^2}{2}$$

$$P_{\text{yanbandlar}} \Rightarrow P_s = \frac{\mu^2 A_c^2}{2} P_{m_n}$$

$$\Leftarrow P_{m_n} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |m_n(t)|^2 dt$$

$$P_y = P_c + P_s \quad \Leftarrow \left( \int m_n(t) dt \approx 0 \right) \text{ Ses işaretleri için.}$$

$$\text{Verim: } V = \frac{P_s}{P_y} = \frac{P_s}{P_c + P_s} = \frac{\frac{\mu^2 A_c^2}{2} P_{m_n}}{\frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{2} P_{m_n}} = \frac{\mu^2 P_{m_n}}{1 + \mu^2 P_{m_n}}$$

Ton Modülasyonunda:  $m(t) = C \cos(2\pi f_m t)$ ,  $m_n(t) = \cos(2\pi f_m t) \Rightarrow P_{m_n} = \frac{1}{2}$

$$V = \frac{\mu^2}{2 + \mu^2} \quad ; \quad \mu \uparrow \Rightarrow V \uparrow \quad \left| \quad 0 < \mu < 1 \Rightarrow V_{\text{maks}} = \frac{1}{3} = \%33.3 \text{ (saf sinüzoidal için)}$$

Pratikte  $\leq \%25$ .

Örnek:  $m(t) = 5 \cos(2\pi f_m t)$ ,  $\mu = \frac{1}{2}$ ,  $A_c = 10 \Rightarrow y(t)$ ,  $P_c$ ,  $P_s$  ve  $V = ?$

$$m_p = \max|m(t)| = 5 \quad \left. \begin{array}{l} m_n(t) = \cos(2\pi f_m t) \\ y(t) = A_c [1 + \mu m_n(t)] \cos(2\pi f_c t) = 10 \left[ 1 + \frac{1}{2} \cos(2\pi f_m t) \right] \cos(2\pi f_c t) \end{array} \right\}$$

$$P_c = \frac{A_c^2}{2} = 50$$

$$P_s = \frac{\mu^2 A_c^2 P_{m_n}}{2} = \frac{(\frac{1}{2})^2 \cdot 10^2 \cdot \frac{1}{2}}{2} = 6,25 \text{ W}$$

$$P_y = P_c + P_s = 56,25 \text{ W}$$

$$V = \frac{P_s}{P_y} = \frac{6,25}{56,25} = \frac{1}{9} \quad \%10 \text{ 'un biraz üzerinde.}$$

\* Anten belli bir  $\Omega$  direnci ile modellenirse  $P$ leri  $R$ 'ye  $(\frac{V^2}{R})$  gerekir. Verimi etkilemez.

Örnek:

$$\mu = 0,85$$

$$P_c, P_s = ?$$

$m(t) = 3 \cos(200\pi t) + \sin(600\pi t)$  işareti  $\rightarrow$  Yalğılıkları yapılabilir, böylece türev alınmadan  $\max|m(t)| = 3+1=4$  olarak alınabilir.  
 $c(t) = \cos(2\pi \cdot 10^5 t)$  yi modüle etmektedir.

$$P_c = \frac{1}{2}, \quad P_s = \frac{\mu^2 A_c^2 P_{m_n}}{2} \rightarrow P_{m_n} \text{ için } m_n(t) \Rightarrow \max|m(t)|$$

$$\max|m(t)| \Rightarrow m'(t) = -600\pi \sin(600\pi t) + 600\pi \cos(600\pi t) = 0$$

$$\cos(600\pi t) = \cos(\frac{\pi}{2} - 200\pi t)$$

$$600\pi t = \frac{\pi}{2} - 200\pi t$$

$$t = \frac{1}{1600}$$

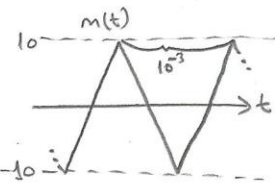
$$m_p = m(\frac{1}{1600}) = 3,6995 \approx 3,7$$

$$m_n(t) = \frac{m(t)}{m_p} = 0,8118 \cos(200\pi t) + 0,2706 \sin(600\pi t)$$

$$P_{m_n} = \frac{1}{2} (0,8118^2 + 0,2706^2) = 0,3661 \text{ Watt.}$$

$$P_s = \frac{\mu^2 A_c^2 P_{m_n}}{2} = 0,1323 \text{ Watt.}$$

Örnek:



$$A_c = 20$$

$$\mu = 0,75$$

a)  $y(t)$  yi çiziniz.

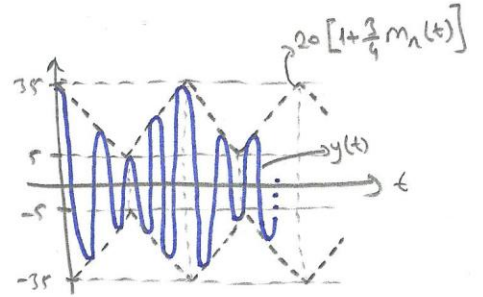
b)  $P_c, P_s, P_y, V = ?$

$$a) y(t) = A_c [1 + \mu m_n(t)] \cos(2\pi f_c t)$$

$$y(t) = 20 \left[ 1 + \frac{3}{4} m_n(t) \right] \cos(2\pi f_c t)$$

$$m_p = \max|m(t)| = 10 \quad m_n(t) = \frac{m(t)}{m_p}$$

$$-1 \leq m_n(t) \leq 1$$

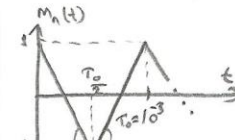


$$b) P_c = \frac{20^2}{2} = 200 \text{ W}, \quad P_s = \frac{\mu^2 A_c^2 P_{m_n}}{2}$$

$$P_s = 37,5 \text{ W}$$

$$P_y = 237,5 \text{ W}$$

$$V = \frac{P_s}{P_y} = \frac{37,5}{237,5} = 0,1575 \approx \%16$$



$$m_1 = \frac{-2}{10^3/2} = -4000$$

$$m_2 = \frac{2}{10^3/2} = 4000$$

$$y = m_1 t + 1 = -4000t + 1$$

$$y = m_2 t + 1 = 4000t - 3$$

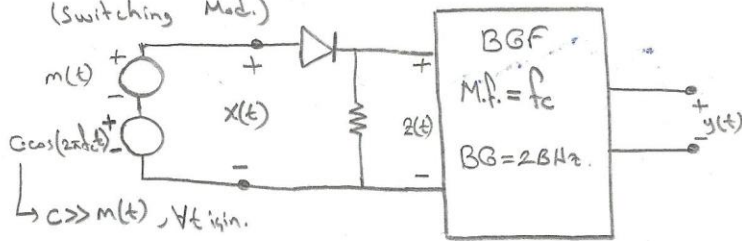
$$P_{m_n} = \frac{1}{T_0} \int_{T_0} |m_n(t)|^2 dt = \frac{1}{10^{-3}} \int_{-10^{-3}/2}^{10^{-3}/2} (-4000t + 1)^2 dt + \frac{1}{10^{-3}} \int_{-10^{-3}/2}^{10^{-3}/2} (4000t - 3)^2 dt$$

$$P_{m_n} = 0,334 \text{ W}$$



**BT - GYB - GM Modulatorları:**

**Anahtarlama Modulatorü (Switching Mod.)**



\* Diyot yerine diyot köprüsü de kullanılabilir. Görevi anahtarlama yapmaktır.

$$x(t) = m(t) + C \cos(2\pi f_c t)$$

$$z(t) = x(t) W(t) = [m(t) + C \cos(2\pi f_c t)] \left[ \frac{1}{2} + \frac{2}{\pi} \left( \cos(2\pi f_c t) - \frac{1}{3} \cos(2\pi 3f_c t) + \frac{1}{5} \cos(2\pi 5f_c t) - \dots \right) \right]$$

$$= \frac{C}{2} \cos(2\pi f_c t) + \frac{2}{\pi} m(t) \cos(2\pi f_c t) + \text{Diğer terimler}$$

→ nfc frekans terimli (n ≥ 2)  
→ 1/2 m(t) × C/π (DC)

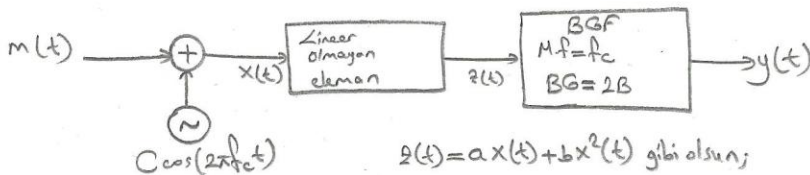
$$y(t) = \frac{C}{2} \cos(2\pi f_c t) + \frac{2}{\pi} m(t) \cos(2\pi f_c t)$$

$$y(t) = \frac{C}{2} \left[ 1 + \frac{4}{\pi C} m(t) \right] \cos(2\pi f_c t)$$

Ac      μ/mp

BGF X

**Lineer Olmayan Modulator (Nonlinear, Power-law mod.)**



z(t) = ax(t) + bx²(t) gibi olsun;

$$x(t) = m(t) + C \cos(2\pi f_c t)$$

$$z(t) = a m(t) + a C \cos(2\pi f_c t) + b [m²(t) + C² \cos²(2\pi f_c t) + 2m(t) C \cos(2\pi f_c t)]$$

$$= a m(t) + a C \cos(2\pi f_c t) + b m²(t) + 2b m(t) C \cos(2\pi f_c t) + \frac{b C²}{2} [1 + \cos(2\pi 2f_c t)]$$

BGF → X      X      X      X

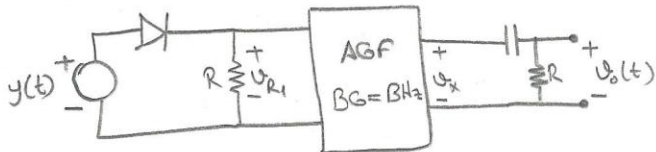
$$y(t) = a C \cos(2\pi f_c t) + 2b m(t) C \cos(2\pi f_c t)$$

$$y(t) = a C \left[ 1 + \frac{2b}{a} m(t) \right] \cos(2\pi f_c t)$$

Ac      μ/mp

**BT - GYB - GM'nin Faz Tahminsiz Demodülasyonu: (Phase Non-coherent)**

**1) Doğrultucu Dedektör (Rectifier Detection)**



$$y(t) = A_c [1 + \mu m_n(t)] \cos(2\pi f_c t)$$

$$V_{R1} = y(t) W(t) = A_c [1 + \mu m_n(t)] \cos(2\pi f_c t) \left[ \frac{1}{2} + \frac{2}{\pi} \left\{ \cos(2\pi f_c t) - \frac{1}{3} \cos(2\pi 3f_c t) + \frac{1}{5} \cos(2\pi 5f_c t) - \dots \right\} \right]$$

$$V_x = \frac{A_c}{\pi} + \frac{\mu A_c}{\pi} m_n(t) + \text{Diğer terimler}$$

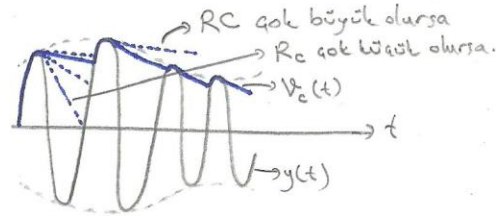
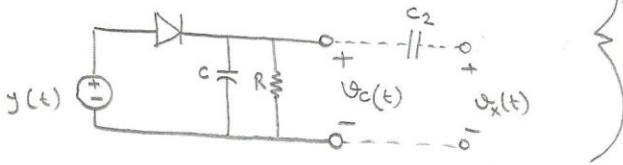
→ nfc frekanslı (n ≥ 1)  
→ m(t) cos(nfc) frekanslı

Kapasitör DCX ⇒ Vo(t) = μAc/π m\_n(t)

AGF X      (41)

## 2) Zarf Dedektörü: (Envelope Detector)

$$BG\{m(t)\} = B \text{ Hz} ; f_c \gg B$$



$RC \gg \frac{1}{f_c}$  olmalı ki tamamen boşalmasın.

$\frac{1}{f_c} \ll RC \ll \frac{1}{B}$  olmalı ki  $V_c(t)$  mesaj işaretini takip edebilsin.



$C_2$  devreye girer  
DC bileşen kaybolur.



$$V_c(t) = \alpha_1 + \alpha_2 m(t) + \text{küçük genlikli } f_c \text{ frekanslı dalgacık (ripple)}$$

↳ DC bileşen.

$$V_x(t) = \alpha_2 m(t) + \text{dalgaçık.}$$

AGF konulursa ripple filtrelenir. B Hz'lik mesaj iletmek için 2B Hz band kullanıyoruz (BG)

## • Dörtlülü Genlik Modülasyonu, (DÖGM)

[Quadrature Amplitude Mod. (QAM)] / Quadrature Multiplexing

Gözüm: Birbirine ortogonal iki işaret kullanmak.

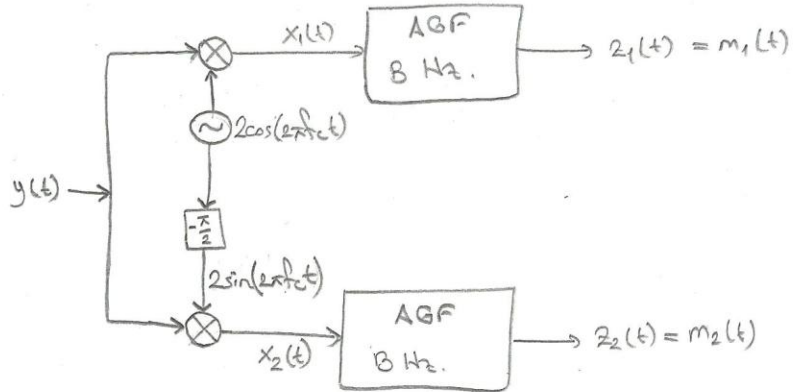
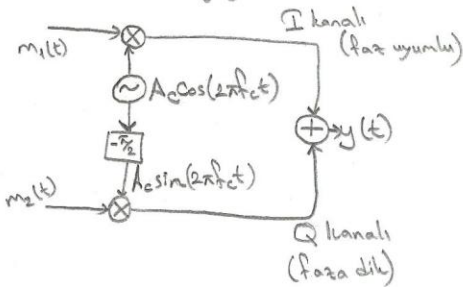
$$y(t) = A_c [m_1(t) \cos(2\pi f_c t) + m_2(t) \sin(2\pi f_c t)] \rightarrow P_y = \frac{A_c^2}{2} P_{m_1} + \frac{A_c^2}{2} P_{m_2}$$

$$BG\{m_1(t)\} = BG\{m_2(t)\} = B \text{ Hz.}$$

↳ ortogonalitenin ötürü bu şekilde olur.

$$\Rightarrow BG\{y(t)\} = 2B \text{ Hz.}$$

Verici Blok Diyagramı



$$\begin{aligned} x_1(t) &= y(t) 2 \cos(2\pi f_c t) \\ &= [m_1(t) \cos(2\pi f_c t) + m_2(t) \sin(2\pi f_c t)] 2 \cos(2\pi f_c t) \\ &= m_1(t) [1 + \cos(4\pi f_c t)] + m_2(t) [0 + \sin(4\pi f_c t)] \\ &= m_1(t) + m_1(t) \cos(4\pi f_c t) + m_2(t) \sin(4\pi f_c t) \end{aligned}$$

$\times \leftarrow \text{AGF} \rightarrow \times$

$$\ast \cos a \cos b = \frac{1}{2} [\cos(a-b) + \cos(a+b)]$$

$$\ast \sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\ast \sin a \cos b = \frac{1}{2} [\sin(a-b) + \sin(a+b)]$$

$$\checkmark z_1(t) = A_c m_1(t)$$

$$\begin{aligned} x_2(t) &= y(t) 2 \sin(2\pi f_c t) \\ &= m_2(t) - m_2(t) \cos(4\pi f_c t) + m_1(t) \sin(4\pi f_c t) \end{aligned}$$

$\times \leftarrow \text{AGF} \rightarrow \times$

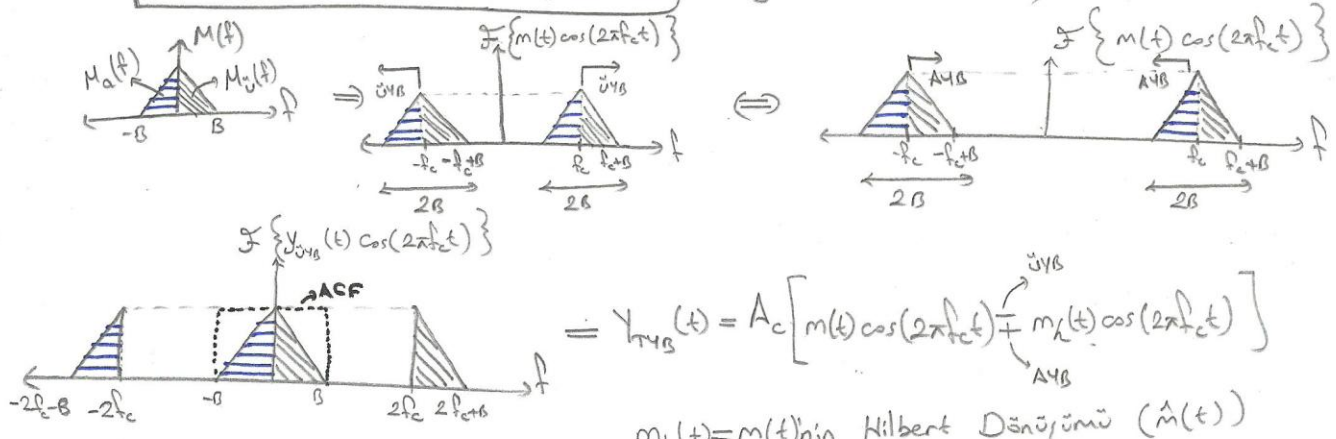
$$\checkmark z_2(t) = A_c m_2(t)$$

$$BG_{\text{verimliliği}} = \frac{BG\{y(t)\}}{\text{mesaj sayısı}} = \frac{2B}{2} = B \text{ Hz.}$$

Bu teknik televizyonlarda renk taşıyan bilgiyi iletmekte kullanılır.

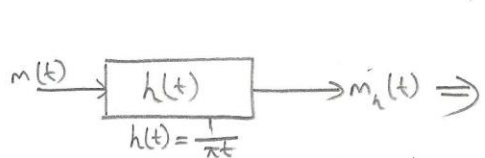
Daha fazla ortogonal işaret ileten teknoloji ADSL modemlerde kullanılır.

**Tek Yan Bantlı (TYB) - GM (Single Side-Band SSB-AM)**



$$Y_{TYB}(t) = A_c [m(t) \cos(2\pi f_c t) \mp m_h(t) \cos(2\pi f_c t)]$$

$m_h(t) = m(t)$ 'nin Hilbert Dönüşümü ( $\hat{m}(t)$ )



$$m_h(t) = m(t) * h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi} \int \frac{m(\tau)}{t-\tau} d\tau$$

$$h(t) = \frac{1}{\pi t} \iff H(f) = ?$$

$$\text{sgn}(t) \iff \frac{1}{j\pi t}$$

$$\frac{1}{j\pi t} \iff \text{sgn}(-f) = -\text{sgn}(f)$$

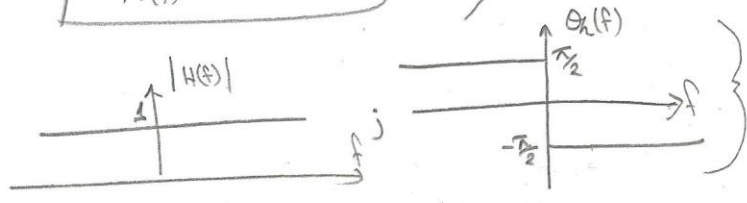
$$\boxed{H(f) = -j \text{sgn}(f)}$$

$$H(f) = -j \text{sgn}(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases} = \begin{cases} e^{-j\pi/2}, & f > 0 \\ e^{j\pi/2}, & f < 0 \end{cases}$$

$$|H(f)| = 1$$

$$\theta_h(f) = \begin{cases} -\pi/2, & f > 0 \\ \pi/2, & f < 0 \end{cases}$$

$$H(f) = |H(f)| e^{j\theta_h(f)}$$



Bu filtre 90°'lik faz farkı üretiyor.

$$P_{TYB} = \frac{A_c^2}{2} P_m \mp \frac{A_c^2}{2} P_{m_h}$$

$\hookrightarrow P_{m_h} = P_m$

$m_h(t) \mid m(t)$   
Genlikleri eşit  
Yalnızca 90°'lik faz farkı var

**Örnek: Ton Modülasyonu.**

$$m(t) = \cos(2\pi f_m t)$$

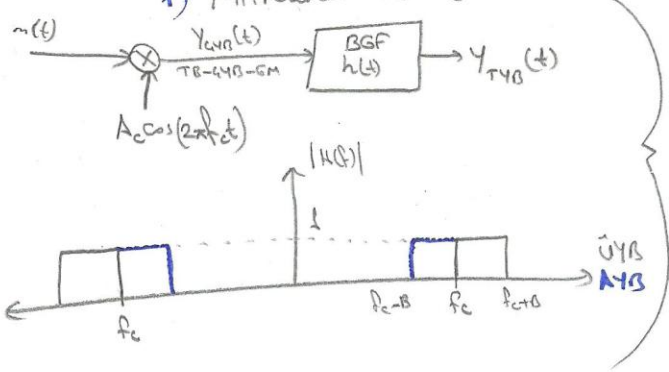
$$m_h(t) = \cos(2\pi f_m t - \pi/2) = \sin(2\pi f_m t)$$

$$Y_{TYB}(t) = A_c \left[ \underbrace{\cos(2\pi f_m t)}_{m(t)} \cos(2\pi f_c t) \mp \underbrace{\sin(2\pi f_m t)}_{m_h(t)} \cos(2\pi f_c t) \right] = A_c \cos(2\pi (f_c \pm f_m) t)$$

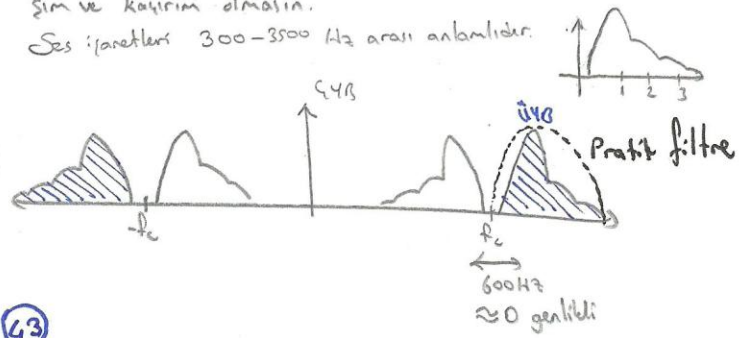
14.05.2012

**TYB-GM Modülörleri:**

1) Filtrelene Bekliği

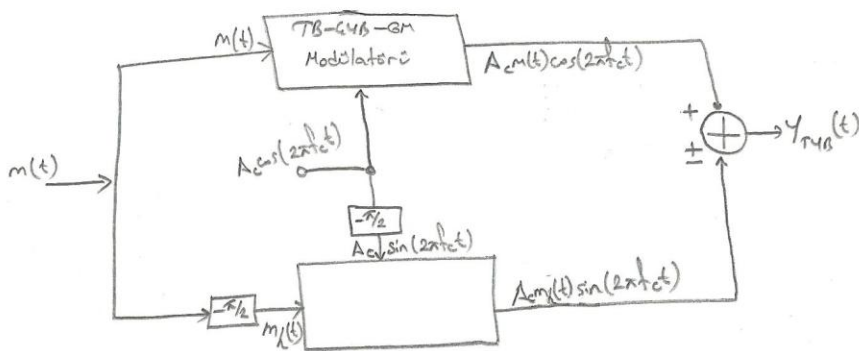


mesaj işaretinin 0 Hz civarında bileşenleri çok zayıf olmalı ki; ÜYB ve LYB iyi birbirinden (pratik filtre ile) koparılabilir karşın ve kayırım olmasın.  
Ses işaretleri 300-3500 Hz arası aralıktadır.



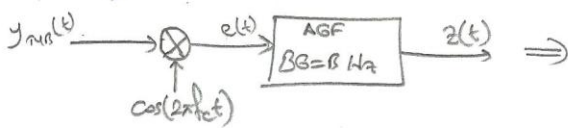


## 2) Faz Öteleme Tekniği (Phase Shifting)



### TYB-GM Demodülatörleri:

#### 1) Faz Tahmini Demodülasyon:



$$y_{TYB}(t) = A_c m(t) \cos(2\pi f_c t) \pm A_c m_h(t) \sin(2\pi f_c t)$$

$$e(t) = y_{TYB}(t) \cos(2\pi f_c t)$$

$$= \frac{A_c}{2} m(t) [1 + \cos(4\pi f_c t)] \pm \frac{A_c}{2} m(t) [0 + \sin(4\pi f_c t)]$$

$$= \frac{A_c}{2} m(t) + \frac{A_c}{2} m(t) \cos(4\pi f_c t) \pm \frac{A_c}{2} m(t) \sin(4\pi f_c t)$$

$\xrightarrow{\text{AGF}}$

$$z(t) = \frac{A_c}{2} m(t)$$

#### 2) Faz Tahminsiz Demodülasyon: (Non-Coherent, Envelope detection of SSB-AM+Carrier)

$$y_{TYB \text{ taşıyıcı}}(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{Büyük Taşıyıcı}} + \underbrace{A [m(t) \cos(2\pi f_c t) \pm m_h(t) \sin(2\pi f_c t)]}_{\text{TYB}}$$

$$= \underbrace{[A_c + A m(t)]}_a \cos(2\pi f_c t) \pm \underbrace{A m_h(t)}_b \sin(2\pi f_c t)$$

$$= E(t) \cos(2\pi f_c t + \theta(t))$$

$$E(t) = \sqrt{[A_c + A m(t)]^2 + [A m_h(t)]^2}^{\frac{1}{2}}$$

$$= [A_c^2 + A^2 m^2(t) + 2A_c A m(t) + A^2 m_h^2(t)]^{\frac{1}{2}}$$

$$= A_c \left[ 1 + \frac{A^2 m^2(t)}{A_c^2} + \frac{2A m(t)}{A_c} + \frac{A^2 m_h^2(t)}{A_c^2} \right]^{\frac{1}{2}}$$

$\approx 0$                        $\approx 0$

$$\begin{cases} a \cos x + b \sin x = c \cos(x + \theta) \\ c = \sqrt{a^2 + b^2} \\ \theta = \arctan\left(\frac{-b}{a}\right) \end{cases}$$

$$A_c \gg A |m(t)| \gg A |m_h(t)| \Rightarrow \cong A_c \left[ 1 + \frac{2A m(t)}{A_c} \right]^{\frac{1}{2}} \cong A_c \left[ 1 + \frac{2A m(t)}{A_c} \cdot \frac{1}{2} \right]$$

$$\Rightarrow E(t) = A_c + A m(t)$$

→ Zarf Dedektörü

Eğer  $|x| \ll 1$  ;  $[1+x]^n \cong [1+nx]$

TYB-GM  $A_c \gg |m(t)|$   
QYB-GM  $A_c \gg |m(t)|$

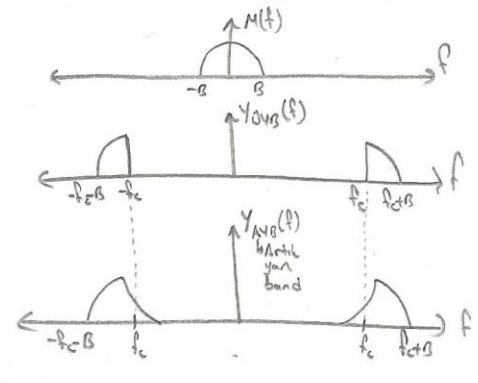
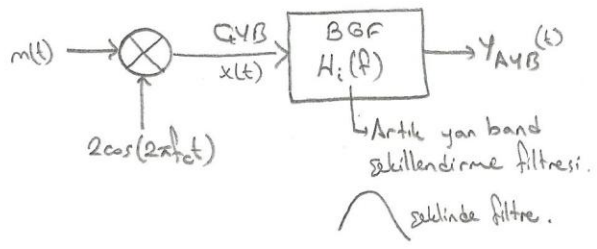
TYB-GM'nin verimi düşük (bedeli, basılan gücün fazla olması); fakat BG'den %50 kazanç vardır.



● Artık Yan Band (AYB) - [Vestigial Sideband AM (VSB-AM)]

$BG\{AYB\} \cong 1,25BG\{TYB\} = 1,25B$

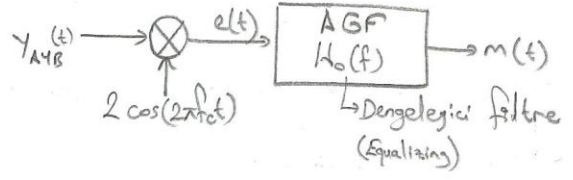
→ Modülasyonun Blok Diyagramı:



$X(f) = M(f-fc) + M(f+fc)$

$Y_{AYB}(f) = X(f) H_i(f) = [M(f-fc) + M(f+fc)] H_i(f)$

→ Demodülasyon (Faz Tahmini Demod.):



$E(f) = Y_{AYB}(f-fc) + Y_{AYB}(f+fc)$

$E(f) \cdot H_o(f) = M(f)$

$[M(f-2fc) H_i(f-fc) + M(f) H_i(f-fc) + M(f) H_i(f+fc) + M(f+2fc) H_i(f+fc)] H_o(f) = M(f)$

$M(f) = M(f) [H_i(f-fc) + H_i(f+fc)] H_o(f)$

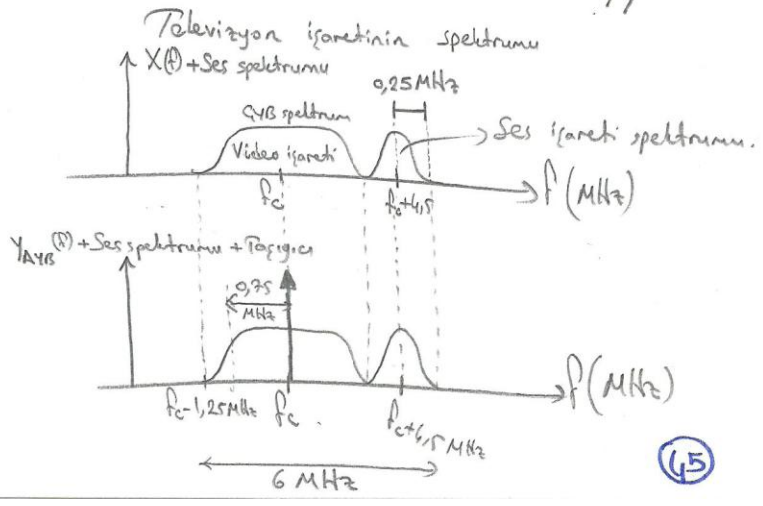
$H_o(f) = \frac{1}{H_i(f-fc) + H_i(f+fc)} ; |f| \leq B$

Eğer  $H_i(f-fc) + H_i(f+fc) = 1$  ise ( $|f| \leq B$ ) ⇒  $H_o(f) = 1 ; |f| \leq B$  olur (ideal AGF)

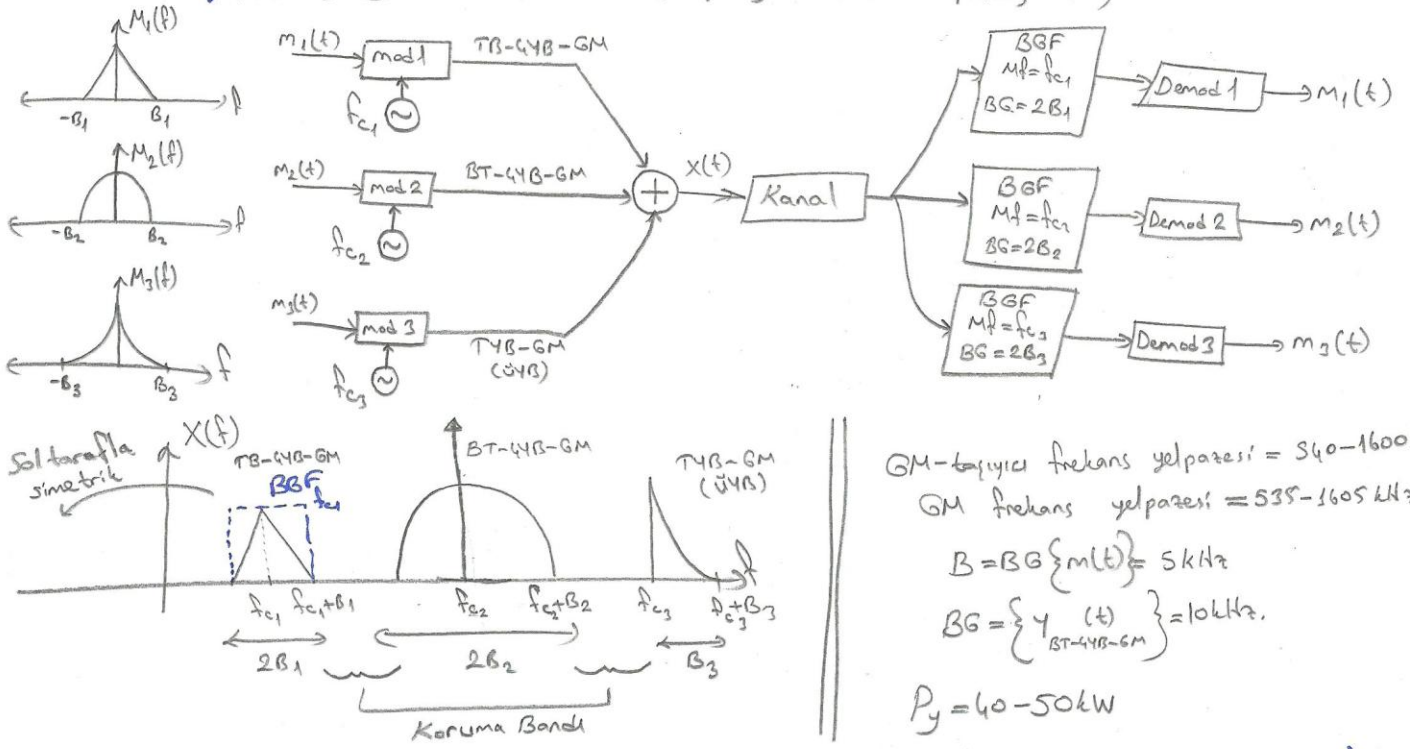
AYB + Taşyıcı = Zarf dedektörü

Taşıyıcı Güç Bölümünden ⇒ TYB > AYB > QYB  
Verimler Bölümünden ⇒ QYB > AYB > TYB

Renk Bandda Siyah-beyaz video işareti  $BG = B = 4.5 \text{ MHz}$   
⇒ AYB,  $B = 6 \text{ MHz}$ .



## → Frekans Bölme: Çözümü: (Frequency Division Multiplexing - FDM)



GM-tarıyıcı frekans yelpazesi = 540-1600kHz

GM frekans yelpazesi = 535-1605kHz

$$B = BG \{m(t)\} = 5 \text{ kHz}$$

$$BG = \left\{ \begin{array}{l} Y(t) \\ \text{BT-4YB-GM} \end{array} \right\} = 10 \text{ kHz}$$

$$P_y = 40-50 \text{ kW}$$

-(Matlab Projesi)-

## # AGİ MODÜLASYONU # (Angle Mod. Exponential Mod. Non-linear Mod.)

Band Geniřlięi (BG) çok yalıdır. Bu da, gürültü bağırsıklığı saęlar (Noise immunity)

Sabit zarlıdır.

$$y(t) = A_c \cos(\theta(t))$$

$$\theta(t) = 2\pi f_c t + \theta_0 \Rightarrow f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

anı frekans  
(instantaneous freq)

$$\theta(t) = 2\pi \int_{-\infty}^t f_i(\alpha) d\alpha$$

→ zamanla deęiřen faz

$$\theta(t) = 2\pi f_c t + \phi(t)$$

$$f_i(t) = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

### 1) Faz Modülasyonu (F2M) / Phase Mod (PM)

$$\phi(t) = k_p m(t)$$

$k_p$ : faz sapma sabiti (phase deviation constant)

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

$$y_{F2M} = A_c \left[ \cos \left( 2\pi f_c t + k_p m(t) \right) \right]$$

$$f_i(t) = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}$$

## 2) Frekans Modülasyonu (FRM) / Freq. Mod. (FM)

$$f_i(t) = f_c + k_f m(t) \quad k_f: \text{frekans sapma sabiti (freq. deviation cons.)}$$

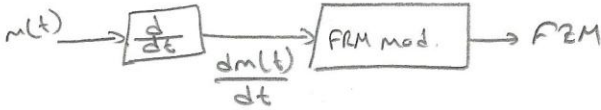
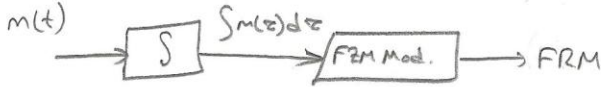
$$\theta(t) = 2\pi \int f_i(\alpha) d\alpha = 2\pi f_c t + 2\pi k_f \int_{-\infty}^t m(\alpha) d\alpha$$

$$y_{FRM}(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_{-\infty}^t m(\tau) d\tau \right]$$

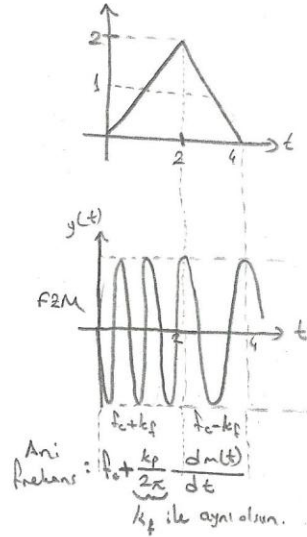
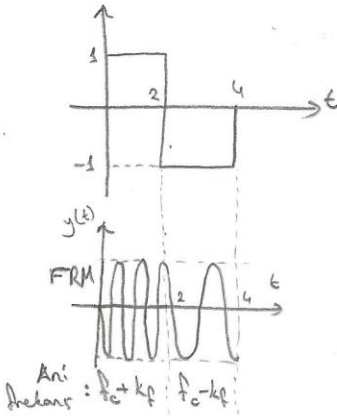
$$y_{a\phi_1}(t) = A_c \cos [2\pi f_c t + \phi(t)]$$

$$\phi(t) = k_p m(t), \text{ F2M (PM)}$$

$$\phi(t) = 2\pi k_f \int_{-\infty}^t m(\tau) d\tau, \text{ FRM (FM)}$$



Örnek:



→ Genel A<sub>ϕ</sub> Modülasyonu:

$$y_{A\phi}(t) = A_c \cos [2\pi f_c t + \phi(t)]$$

$$\phi(t) = \int_{-\infty}^t m(\tau) h(t-\tau) d\tau$$

$$m(t) \rightarrow h(t) \rightarrow \phi(t)$$

$$h(t) = k_p \delta(t) \Rightarrow \text{F2M}$$

$$h(t) = k_f u(t) \Rightarrow \text{FRM}$$

- Açı modülasyonu -

$$y(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

$$\phi(t) = k_f m(t), \text{ F2M}$$

$$\phi(t) = 2\pi k_f \int_{-\infty}^t m(\tau) d\tau, \text{ FRM}$$

$$\text{Ani frekans: } f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$f_i(t) = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$\text{FRM: } f_i(t) = f_c + k_f m(t)$$

$$\text{Güç: } P_y = \frac{A_c^2}{2}$$

• Band Genişliği: BG

$$y(t) = \text{Re}[\hat{y}(t)], \quad \hat{y}(t) = A_c e^{j(2\pi f_c t + \phi(t))}$$

$$= A_c e^{j2\pi f_c t} \cdot e^{j\phi(t)}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{j\phi(t)} = 1 + j\phi(t) - \frac{\phi^2(t)}{2!} + \dots + \frac{j^n \phi^n(t)}{n!} + \dots$$

$$\hat{y}(t) = A_c [\cos(2\pi f_c t) + j\sin(2\pi f_c t)] e^{j\phi(t)}$$

$$y(t) = \text{Re}[\hat{y}(t)] = A_c \left[ \cos(2\pi f_c t) - \phi(t) \sin(2\pi f_c t) - \frac{\phi^2(t)}{2!} \cos(2\pi f_c t) + \dots \right]$$

$$\text{BG}\{m(t)\} = B \text{ Hz ise } \Rightarrow \text{BG}\{\phi(t)\} = B \text{ Hz} \Rightarrow \text{BG}\{\phi^n(t)\} = n \cdot B \text{ Hz}$$

$$\Rightarrow \text{BG}\{y(t)\} = \infty \quad \underline{\underline{}} \quad \text{!}$$

• Efektif Band Genişliği

Carson kuralı;

$$B = \text{BG}\{m(t)\} \quad \text{BG} = 2B(\beta + 1)$$

↳ Sapma oranı (deviation ratio)

$$\rightarrow \text{FRM: } \beta = \frac{\Delta f}{B} \quad \text{→ taşıyıcıdan azami sapma miktarı (max. carrier freq deviation)}$$

$$\Rightarrow \Delta f = k_f \cdot m_p \text{ Hz.} \quad m_p = \text{maks}|m(t)|$$

$$f_i(t) = f_c + k_f m(t)$$

→ F2M:

$$f_i(t) = f_c + \frac{k_f}{2\pi} \frac{dm(t)}{dt} \Rightarrow \Delta f = \frac{k_f}{2\pi} m_p', \quad m_p' = \text{maks}|m'(t)|$$

$$\theta(t) = 2\pi f_c t + \phi(t) \quad ; \quad \Delta\phi = \text{maks}|\phi(t)| : \text{azami faz sapma miktarı}$$

↳ faz

$$\text{F2M: } \Delta\phi = k_p \text{ maks}|m(t)|$$

$$\text{FRM: } \Delta\phi = 2\pi k_f \text{ maks} \left| \int m(\tau) d\tau \right|$$

Örnek:

$$m(t) = 10 \text{ sinc}(10^4 t), \quad k_f = 4000 \quad \text{FRM'nin BG'ni bulunuz.}$$

$$M(f) = 10^{-3} \text{ rect}\left(\frac{f}{10^4}\right) \quad \text{→ } B = 5 \text{ kHz}$$

$$\Delta f = k_f m_p = 4000 \cdot 10 = 4 \cdot 10^4$$

$$\beta = \frac{\Delta f}{B} = \frac{4 \cdot 10^4}{5 \cdot 10^3} = 8$$

$$\text{BG} = 2B(\beta + 1) = 2 \times 5 \text{ kHz} \times 9 = 90 \text{ kHz.}$$



# Dar Bandlı Ağı Modülasyonu (Narrow-Band Angle Mod.)

$$y(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

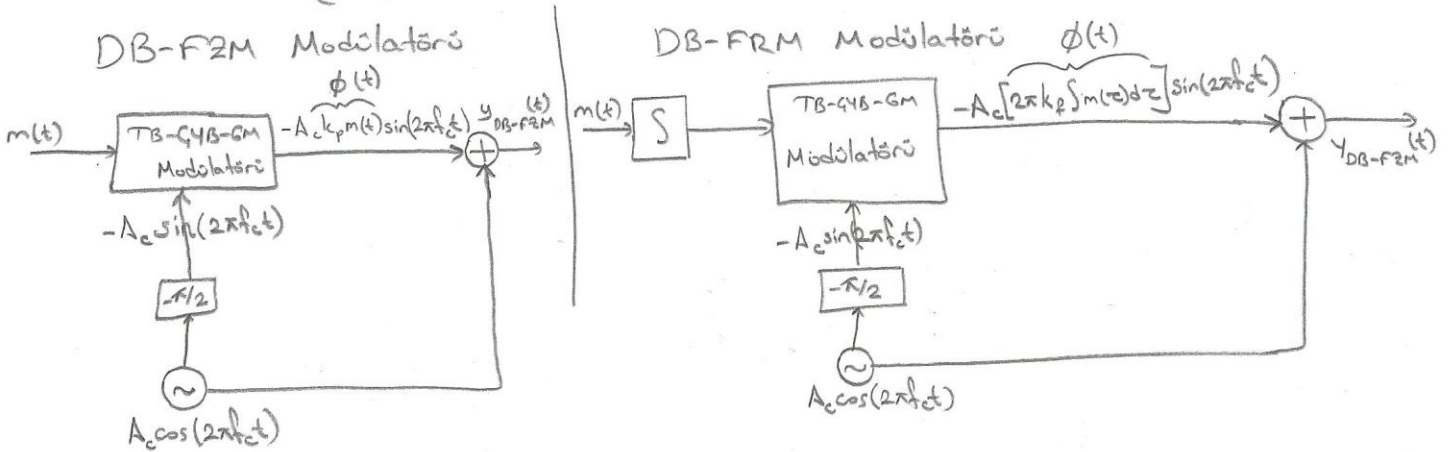
$$= A_c [\cos(2\pi f_c t) \cos(\phi(t)) - \sin(2\pi f_c t) \sin(\phi(t))]$$

$$k_f \ll 1 \Rightarrow \phi(t) \ll 1 \Rightarrow \cos(\phi(t)) \approx 1$$

$$k_p \ll 1 \Rightarrow \sin(\phi(t)) \approx \phi(t)$$

$$\Rightarrow y(t) = A_c [\cos(2\pi f_c t) - \phi(t) \sin(2\pi f_c t)] \quad (\text{Dar bandlı ağı mod.})$$

$$BG\{y(t)\} = 2B \text{ Hz.}$$



$$\rightarrow BG = 2B(\beta + 1)$$

$$k_f \ll 1 \Rightarrow \Delta f = k_f m_p \ll 1 \Rightarrow \beta = \frac{\Delta f}{B} \ll 1 \Rightarrow BG \approx 2B$$

$$k_p \ll 1 \Rightarrow \Delta f = \frac{k_p}{2\pi} m_p' \ll 1 \Rightarrow \beta = \frac{\Delta f}{B} \ll 1 \Rightarrow BG \approx 2B$$

Geniş Bandlı Ağı Mod. (Wideband)

$$k_f \gg 1 \Rightarrow \Delta f \gg 1 \Rightarrow \beta \gg 1 \Rightarrow BG \approx 2B\beta \approx 2\Delta f$$

$$k_p \gg 1 \Rightarrow \Delta f \gg 1 \Rightarrow \beta \gg 1 \Rightarrow BG \approx 2B\beta \approx 2\Delta f$$

Carson Kuralı:  $BS = 2B(\beta + 1) = 2(\beta B + B)$   
 $= 2(\Delta f + B)$

Örnek:  $m(t) = a \cos(2\pi f_m t)$  Ton Mod. (Saf Sinüzoidal)  
 $c(t) = A_c \cos(2\pi f_c t)$

$$F2M: \phi(t) = k_f m(t) = \underbrace{k_f \cdot a}_{:=\beta} \cos(2\pi f_m t) \Rightarrow y_{F2M} = A \cos(2\pi f_c t + \phi(t))$$

$$= A_c \cos(2\pi f_c t + \beta \cos(2\pi f_m t))$$

$$FRM: \phi(t) = 2\pi k_p \int_{-\infty}^t m(\tau) d\tau = 2\pi k_p \frac{a \sin(2\pi f_m t)}{2\pi f_m} = \frac{k_p \cdot a}{f_m} \sin(2\pi f_m t)$$

$$:= \beta$$

$$y_{FRM}(t) = A_c \cos(2\pi f_c t + \phi(t))$$

$$= A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

Bin Sinüzoidal (mesaj) işaretinin açılı mod.

$$m(t) = a \cos(2\pi f_m t) \Rightarrow y_{FRM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)), \quad \beta = \frac{k_p a}{f_m}$$

$$m(t) = a \sin(2\pi f_m t) \Rightarrow y_{FRM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)), \quad \beta = k_p \cdot a$$

$$y(t) = \text{Re}[\hat{y}(t)] \Rightarrow \hat{y}(t) = A_c e^{j(2\pi f_c t + \beta \sin(2\pi f_m t))}$$

$$= A_c e^{j2\pi f_c t} \cdot \underbrace{e^{j\beta \sin(2\pi f_m t)}}_{x(t) \text{ olsun.}}$$

$$x(t) = e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi n f_m t} \quad (T_m = \frac{1}{f_m} \text{ ile periyodik})$$

$$x_n = \frac{1}{T_m} \int_0^{T_m} x(t) e^{-j2\pi n f_m t} dt = \frac{1}{T_m} \int_0^{T_m} e^{j\beta \sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt$$

$$x_n = \frac{1}{T_m} \int_0^{2\pi} e^{j\beta \sin \alpha} e^{-jn\alpha} \frac{d\alpha}{2\pi f_m} = \frac{1}{2\pi} \int_0^{2\pi} e^{j(\beta \sin \alpha - n\alpha)} d\alpha$$

$$\left[ \begin{array}{l} 2\pi f_m t = \alpha \\ dt = \frac{d\alpha}{2\pi f_m} \end{array} \right]$$

$$= J_n(\beta) \quad n. \text{ mertebeden 1. tür Bessel fonksiyonu.}$$

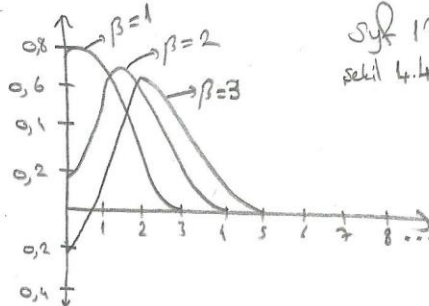
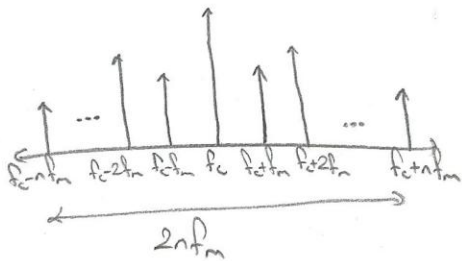
$$x(t) = e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

$$\hat{y}(t) = A_c e^{j2\pi f_c t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

$$\Rightarrow y(t) = \text{Re}[\hat{y}(t)] = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

$$J_{-n}(\beta) = (-1)^n J_n(\beta)$$

$$|J_{-n}(\beta)| = |J_n(\beta)|$$



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$$n > \beta + 1 \Rightarrow J_n(\beta) \ll \Rightarrow BG = 2n f_m \approx 2(\beta + 1) f_m \approx 2 f_m (\beta + 1)$$

$$B = BG \{m(t)\} = f_m$$